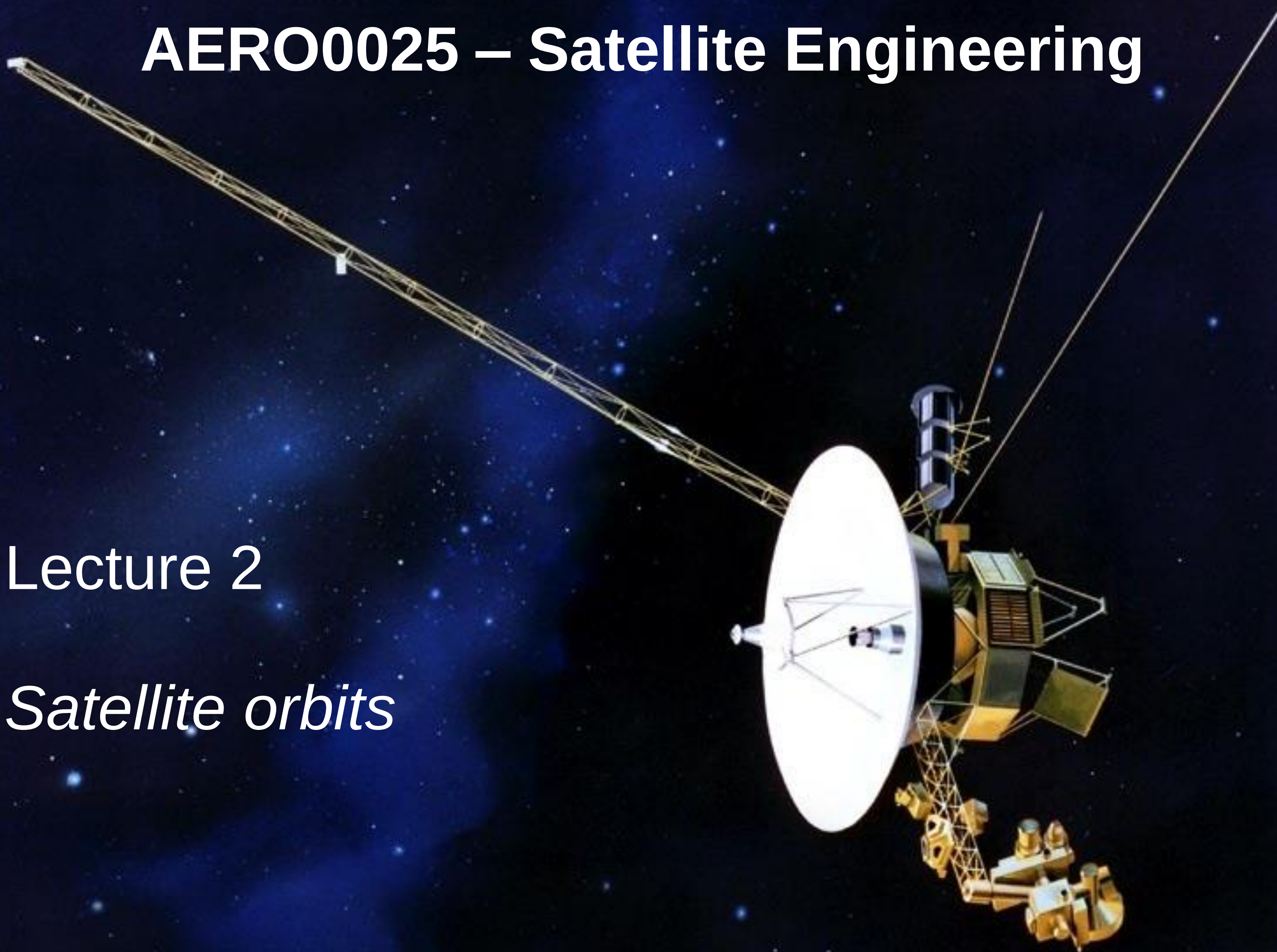
AERO0025 – Satellite Engineering



Lecture 2

Satellite orbits

Can the Orbit Affect …

Mass of the satellite ?

Power generation ?

Space radiation environment ?

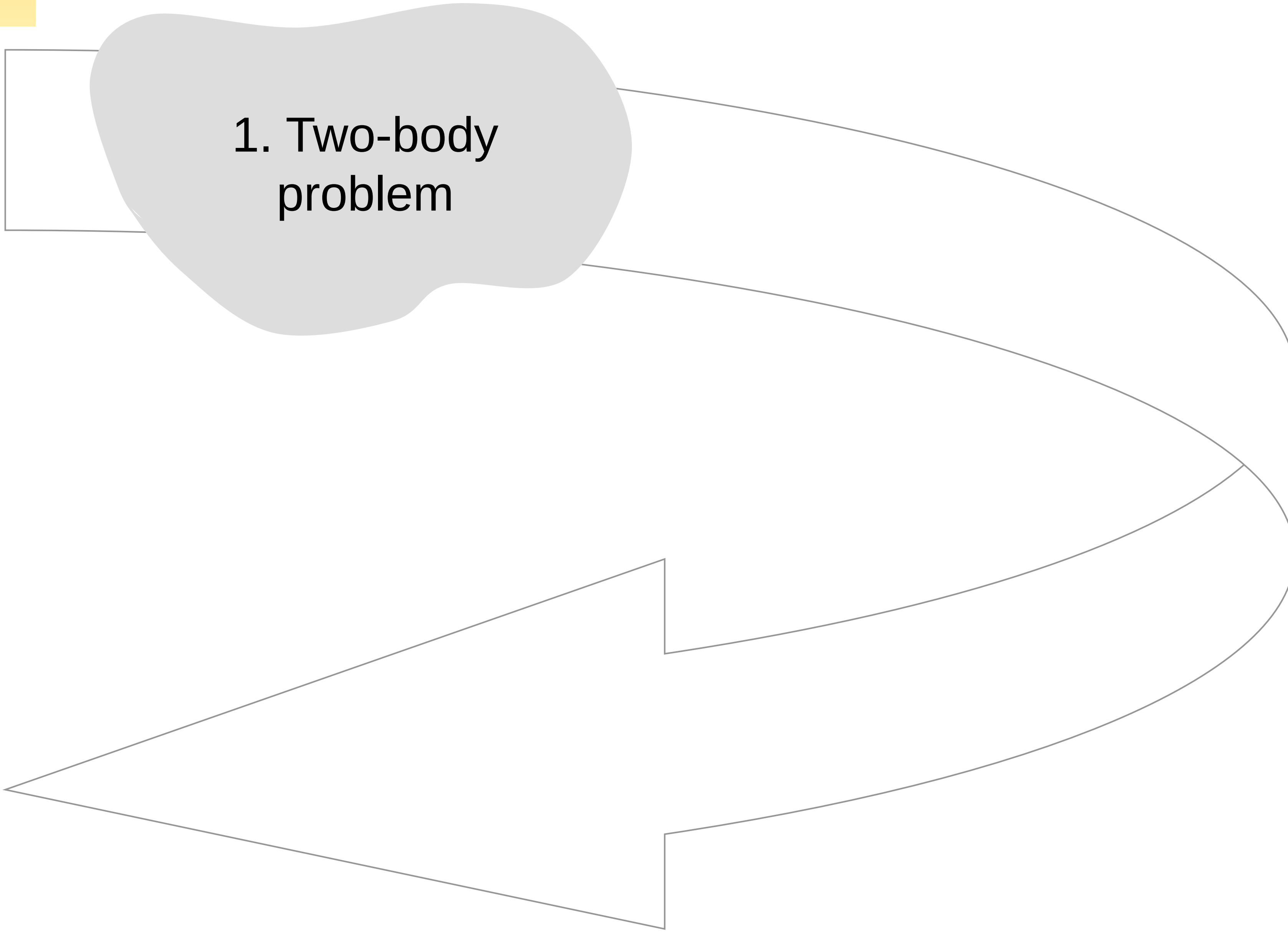
Revisit time of satellite to a point on Earth ?

Thermal control ?

Launch costs ?

YES !

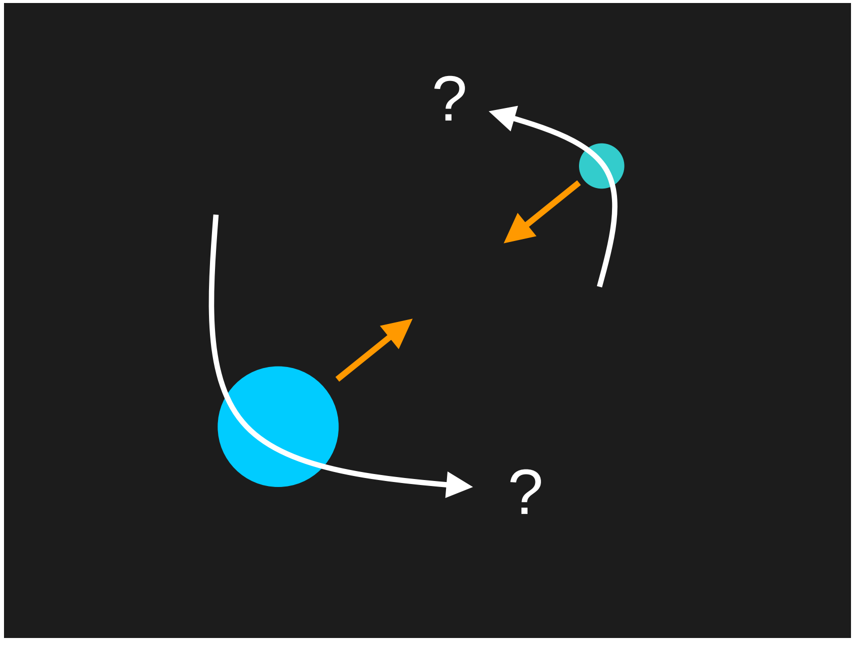
Satellite Orbits



1. Definition of the 2-Body Problem

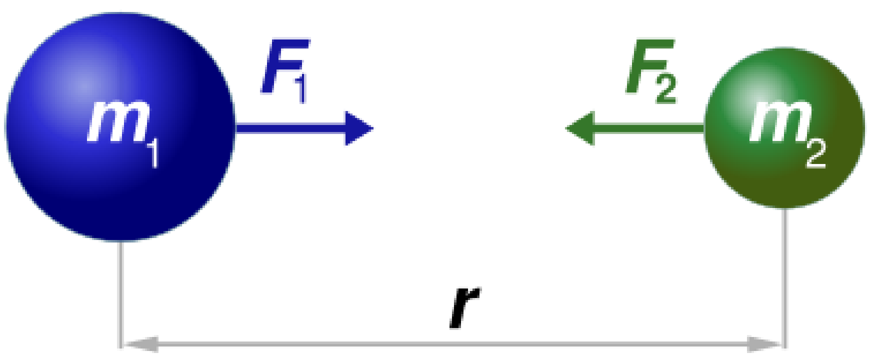
Motion of two bodies due solely to their own mutual gravitational attraction. Also known as **Kepler problem**.

Assumption: two point masses (or equivalently spherically symmetric objects).



1. Gravitational Force

Every point mass attracts every other point mass by a force pointing along the line intersecting both points. The force is proportional to the product of the two masses and inversely proportional to the square of the distance between the point masses:



$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

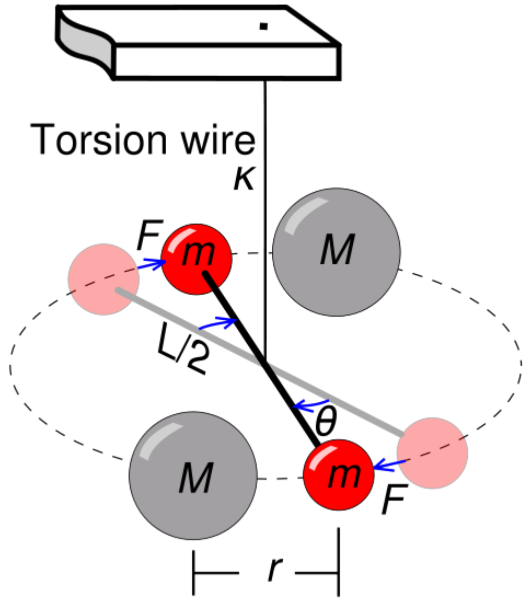
Gravitational Constant

By measuring the mutual attraction of two bodies of known mass, the gravitational constant *G* can directly be determined from torsion balance experiments.

Due to the small size of the gravitational force, *G* is presently only known with limited accuracy and was first determined many years after Newton’s discovery:

$$(6.67428 \pm 0.00067) \times 10^{-11}\ \mathrm{m^3.kg^{-1}.s^{-2}}$$

(http://www.physics.nist.gov/cgi-bin/cuu/Value?bg)



$$k\theta = L\frac{GmM}{r^2}$$

1. Gravitational Parameter of a Body

$$\mu = GM_{\oplus}$$

The gravitational parameter of the Earth has been determined with considerable precision from the analysis of laser distance measurements of artificial satellites:

$$398600.4418 \pm 0.0008 \text{ km}^3.\text{s}^{-2}.$$

The uncertainty is 1 to 5e8, much smaller than the uncertainties in *G* and *M* separately (~1 to 1e4 each).

2-Body Problem: Governing Equations

Newton's second law:

$F=ma$ where F is the gravitational force

2-Body Problem: Governing Equations

Newton's second law:

F=ma where *F* is the gravitational force

What did Richard Feynman mean about the Second Law of Motion? Where was the error?

JANUARY 17, 2021 / FRANCES48 / 0 COMMENTS

Richard Feynman writes about Newton's Second Law of Motion in his work "Lectures on Physics" (Chapter 15):

„For over 200 years the equations of motion enunciated by Newton were believed to describe nature correctly, and the first time that an error in these laws was discovered, the way to correct it was also discovered. Both the error and its correction were discovered by Einstein in 1905.

2-Body Problem: Governing Equations

Newton's Second Law, which we have expressed by the equation

$$F = d(mv)/dt$$

was stated with the tacit assumption that m is a constant, but we now know that this is not true, and that the mass of a body increases with velocity. In Einstein's corrected formula m has the value

$$m = \frac{m_0}{\sqrt{1 - v^2 \,/\, c^2}}$$

where the rest mass represents the mass of a body that is not moving and c is the speed of light [...].

Newton's law is still an excellent approximation of the effects of gravity if:

$$\frac{\Phi}{c^2} = \frac{GM}{rc^2} <<< 1, \text{ and } \left(\frac{v}{c}\right)^2 <<< 1$$

General Relativity: Earth-Sun Example

$$\frac{\Phi}{c^2} = \frac{GM_{sun}}{r_{orbit}c^2} \sim 10^{-8}, \text{ and } \left(\frac{v}{c}\right)^2 = \left(\frac{2\pi r_{orbit}}{1 \text{ year}.c}\right)^2 \sim 10^{-8}$$

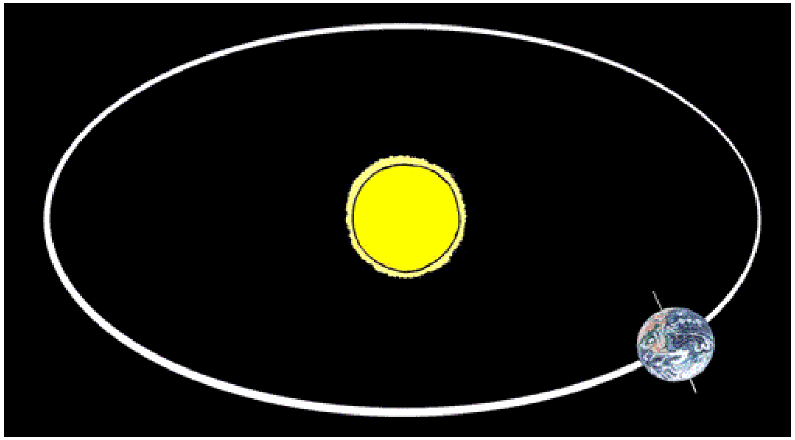
OK !

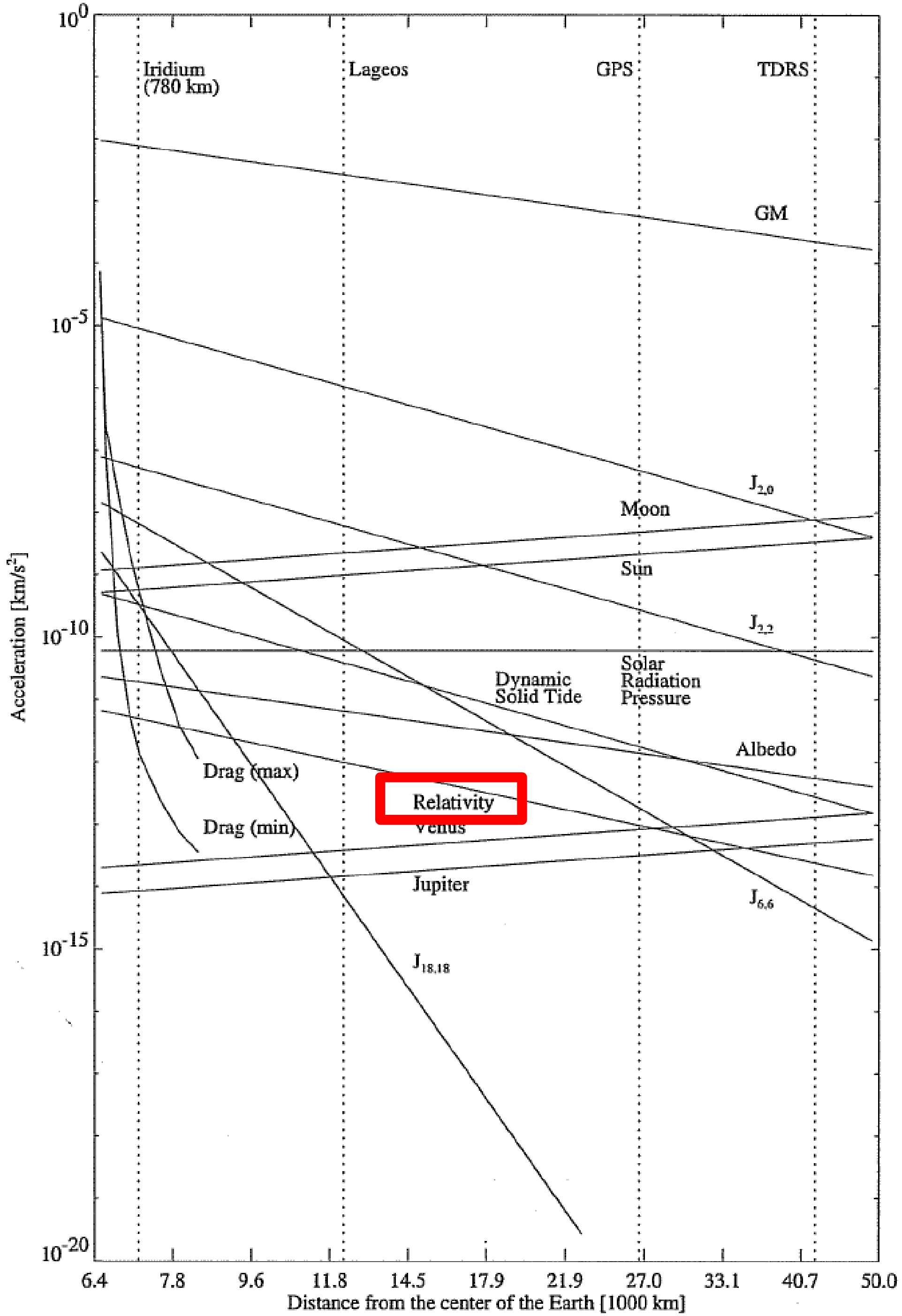
$G=6.67428 \times 10^{-11}$ m^3.kg^{-1}.s^{-2}

$r_{orbit}=1.5 \times 10^{11}$ m (1 AU)

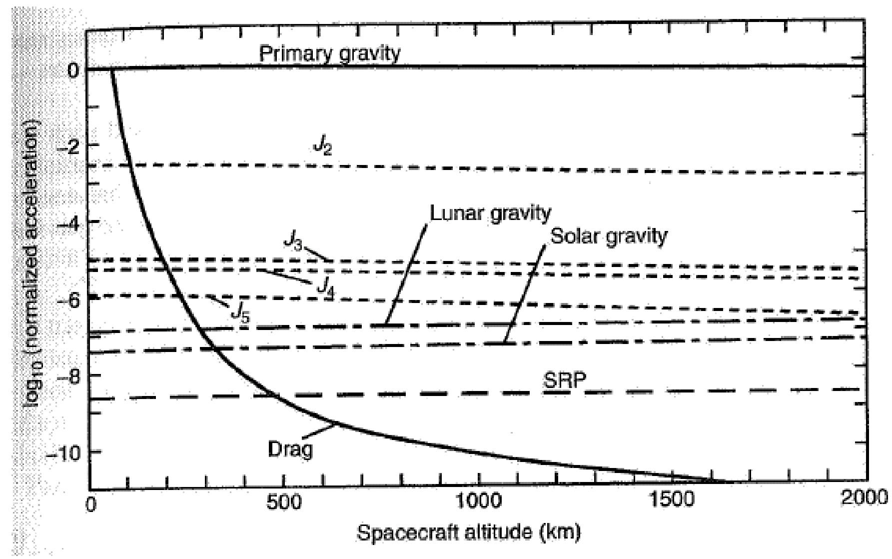
$M_{sun}=1.9891 \times 10^{30}$ kg

c=3e8 m.s^{-1}





Satellite dependent !



Montenbruck and Gill, *Satellite orbits,* Springer, 2000

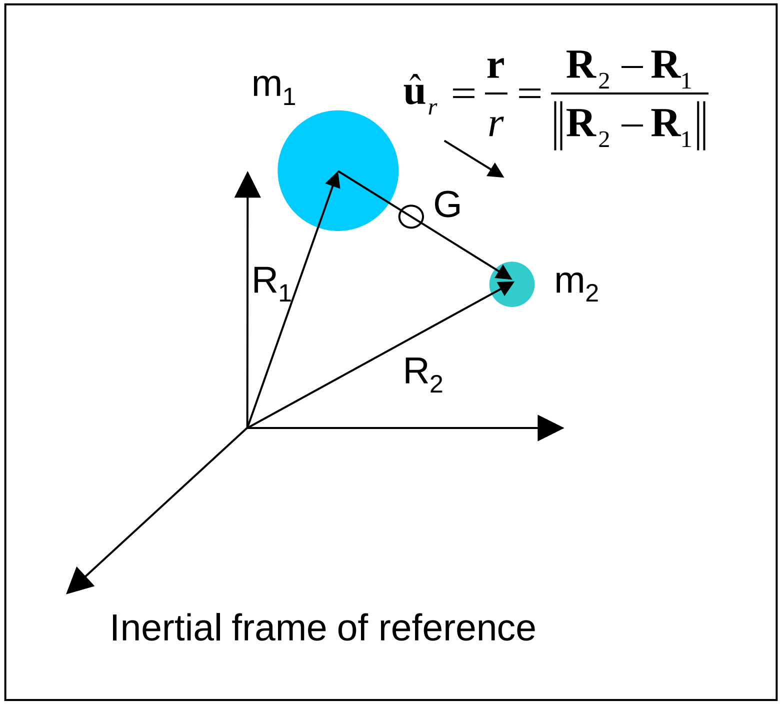
Fortescue et al., *Spacecraft systems engineering*, 2003

1. Motion of the Two Bodies

$$m_1\ddot{\mathbf{R}}_1 = \frac{Gm_1m_2}{r^2}\hat{\mathbf{u}}_r$$

$$+$$

$$m_2\ddot{\mathbf{R}}_2 = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{u}}_r$$



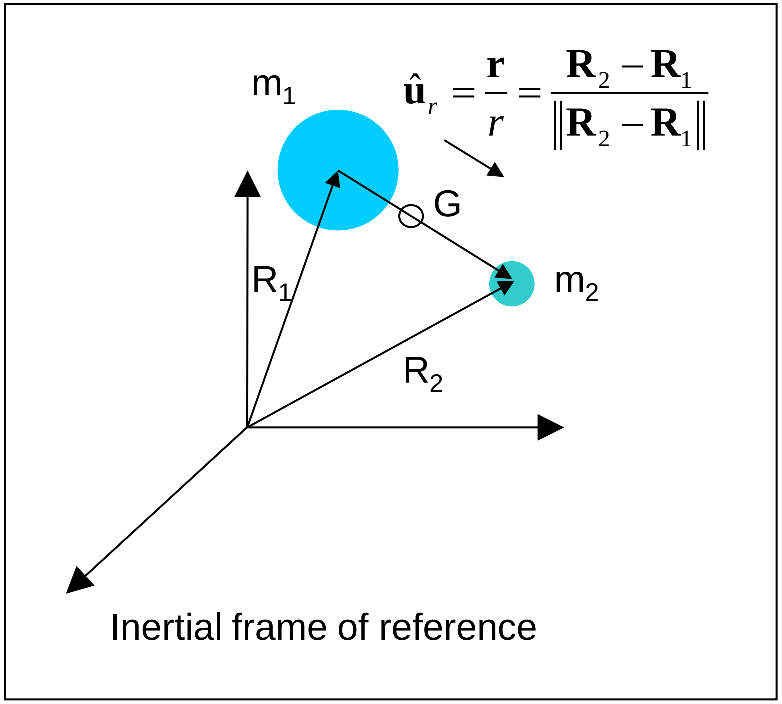
1. Equations of Relative Motion

$$-m_1 m_2 \ddot{\mathbf{R}}_1 = \frac{-G m_1 m_2^2}{r^2}\hat{\mathbf{u}}_r$$

$$+$$

$$m_1 m_2 \ddot{\mathbf{R}}_2 = -\frac{G m_1^2 m_2}{r^2}\hat{\mathbf{u}}_r$$

$$\ddot{\mathbf{R}}_2 - \ddot{\mathbf{R}}_1 = -\frac{G(m_1 + m_2)}{r^2}\hat{\mathbf{u}}_r$$



$$\boxed{\ddot{\mathbf{r}} = -\frac{\mu}{r^3}\mathbf{r}}$$

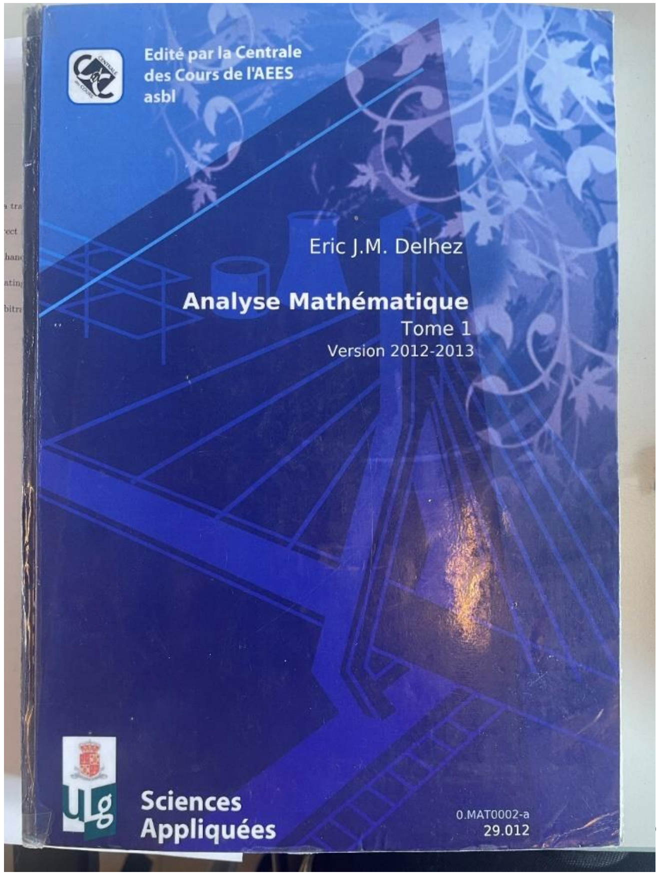
μ is the gravitational parameter

The motion of m_2 as seen from m_1 is the same as the motion of m_1 as seen from m_2.

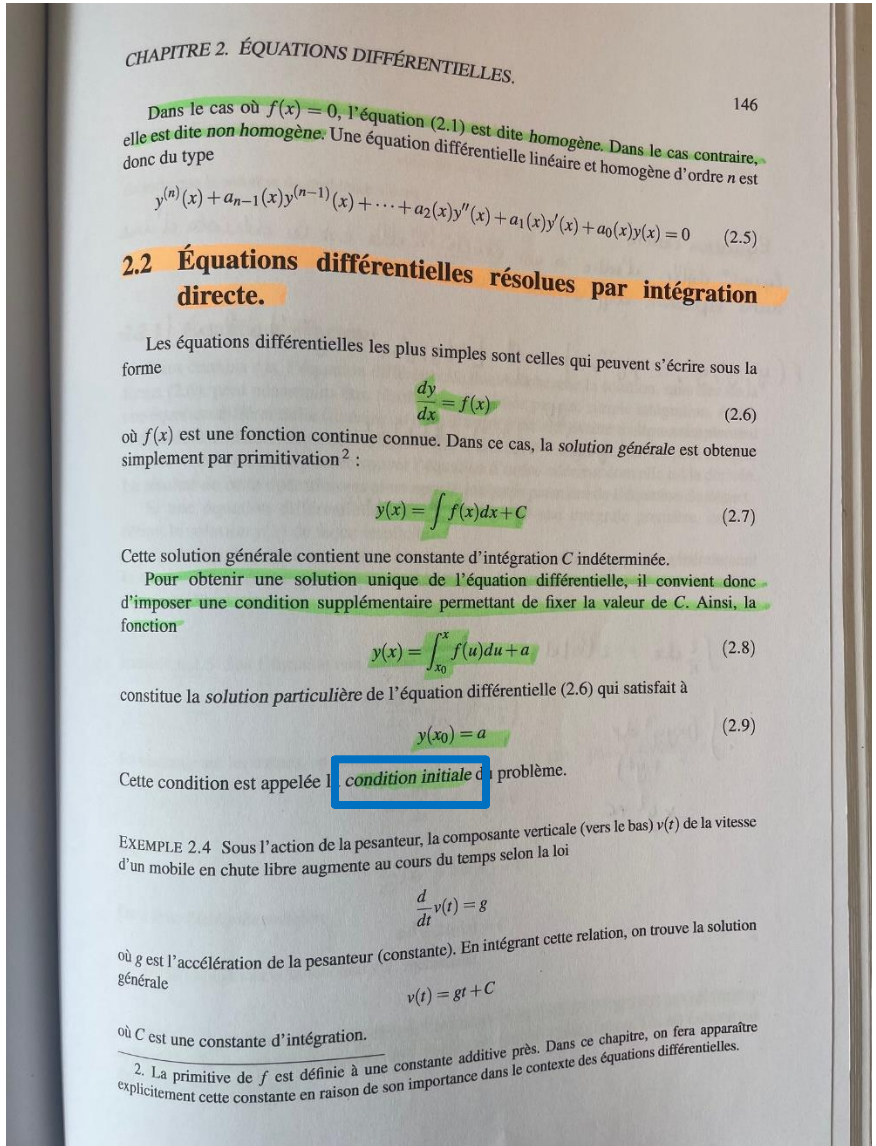
1. Equations of Relative Motion

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3}\mathbf{r}$$

How to solve it and find $\mathbf{r} = \mathbf{r}(t)$?



1. How many initial conditions ?



Dans le cas où $f(x) = 0$, l'équation (2.1) est dite *homogène*. Dans le cas contraire, elle est dite *non homogène*. Une équation différentielle linéaire et homogène d'ordre n est donc du type

$$y^{(n)}(x) + a_{n-1}(x)y^{(n-1)}(x) + \cdots + a_2(x)y''(x) + a_1(x)y'(x) + a_0(x)y(x) = 0 \qquad (2.5)$$

2.2 Équations différentielles résolues par intégration directe.

Les équations différentielles les plus simples sont celles qui peuvent s'écrire sous la forme

$$\frac{dy}{dx} = f(x) \qquad (2.6)$$

où $f(x)$ est une fonction continue connue. Dans ce cas, la *solution générale* est obtenue simplement par primitivation[2] :

$$y(x) = \int f(x)dx + C \qquad (2.7)$$

Cette solution générale contient une constante d'intégration C indéterminée.

Pour obtenir une solution unique de l'équation différentielle, il convient donc d'imposer une condition supplémentaire permettant de fixer la valeur de C. Ainsi, la fonction

$$y(x) = \int_{x_0}^{x} f(u)du + a \qquad (2.8)$$

constitue la *solution particulière* de l'équation différentielle (2.6) qui satisfait à

$$y(x_0) = a \qquad (2.9)$$

Cette condition est appelée la *condition initiale* du problème.

EXEMPLE 2.4 Sous l'action de la pesanteur, la composante verticale (vers le bas) $v(t)$ de la vitesse d'un mobile en chute libre augmente au cours du temps selon la loi

$$\frac{d}{dt}v(t) = g$$

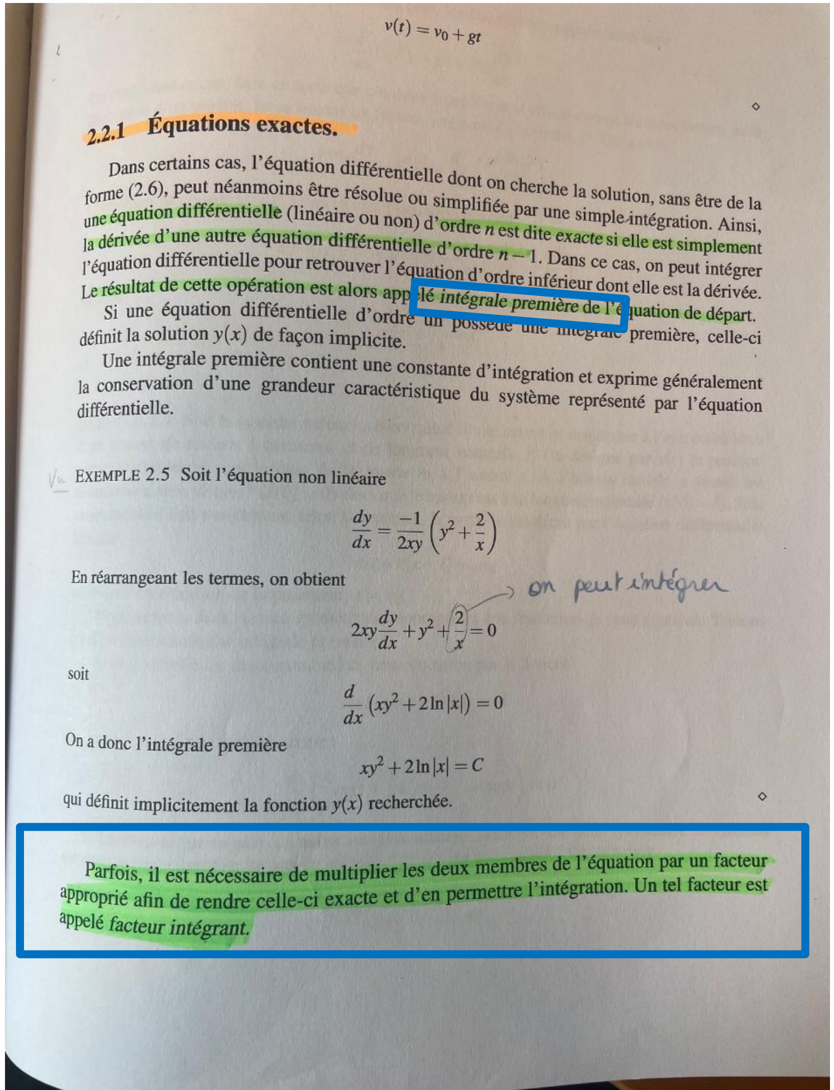
où g est l'accélération de la pesanteur (constante). En intégrant cette relation, on trouve la solution générale

$$v(t) = gt + C$$

où C est une constante d'intégration.

2. La primitive de f est définie à une constante additive près. Dans ce chapitre, on fera apparaître explicitement cette constante en raison de son importance dans le contexte des équations différentielles.

1. Find constants of the motion



$$v(t) = v_0 + gt$$

2.2.1 Équations exactes.

Dans certains cas, l'équation différentielle dont on cherche la solution, sans être de la forme (2.6), peut néanmoins être résolue ou simplifiée par une simple intégration. Ainsi, une équation différentielle (linéaire ou non) d'ordre n est dite *exacte* si elle est simplement la dérivée d'une autre équation différentielle d'ordre $n-1$. Dans ce cas, on peut intégrer l'équation différentielle pour retrouver l'équation d'ordre inférieur dont elle est la dérivée. Le résultat de cette opération est alors appelé *intégrale première* de l'équation de départ.

Si une équation différentielle d'ordre un possède une intégrale première, celle-ci définit la solution $y(x)$ de façon implicite.

Une intégrale première contient une constante d'intégration et exprime généralement la conservation d'une grandeur caractéristique du système représenté par l'équation différentielle.

EXEMPLE 2.5 Soit l'équation non linéaire

$$\frac{dy}{dx} = \frac{-1}{2xy}\left(y^2 + \frac{2}{x}\right)$$

En réarrangeant les termes, on obtient

$$2xy\frac{dy}{dx} + y^2 + \frac{2}{x} = 0$$

on peut intégrer

soit

$$\frac{d}{dx}\left(xy^2 + 2\ln|x|\right) = 0$$

On a donc l'intégrale première

$$xy^2 + 2\ln|x| = C$$

qui définit implicitement la fonction $y(x)$ recherchée. ◇

Parfois, il est nécessaire de multiplier les deux membres de l'équation par un facteur approprié afin de rendre celle-ci exacte et d'en permettre l'intégration. Un tel facteur est appelé *facteur intégrant*.

1. Constant Angular Momentum

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3}\mathbf{r} \quad \xrightarrow{\mathbf{r}\times} \quad \mathbf{r}\times\ddot{\mathbf{r}} = \mathbf{r}\times\left(-\frac{\mu}{r^3}\mathbf{r}\right) = 0$$

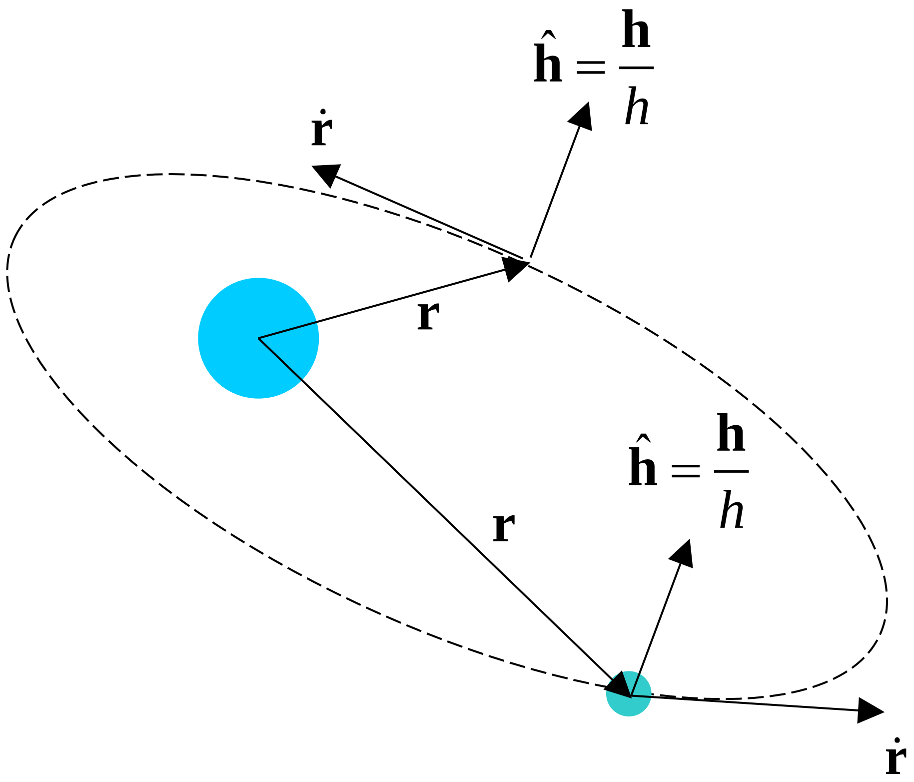
$$\mathbf{h} = \mathbf{r}\times\dot{\mathbf{r}} \quad \xrightarrow{d/dt} \quad \frac{d\mathbf{h}}{dt} = \dot{\mathbf{r}}\times\dot{\mathbf{r}} + \mathbf{r}\times\ddot{\mathbf{r}} = \mathbf{r}\times\ddot{\mathbf{r}}$$

Specific angular momentum (rotational analog of linear momentum)

$$\Downarrow$$

$$\boxed{\frac{d\mathbf{h}}{dt} = 0 \rightarrow \mathbf{r}\times\dot{\mathbf{r}} = \text{constant}=\mathbf{h}}$$

1. The Motion Lies in a Fixed Plane



The fixed plane is the **orbit plane** and is normal to the angular momentum vector.

$$\mathbf{r} \times \dot{\mathbf{r}} = \text{constant} = \mathbf{h}$$

1. First Integral of Motion

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3}\mathbf{r} \quad \xRightarrow{\times \mathbf{h}} \quad \ddot{\mathbf{r}} \times \mathbf{h} = -\frac{\mu}{r^3}\mathbf{r} \times \mathbf{h} = -\frac{\mu}{r^3}\mathbf{r} \times (\mathbf{r} \times \dot{\mathbf{r}})$$

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b}(\mathbf{a}.\mathbf{c}) - \mathbf{c}(\mathbf{a}.\mathbf{b}) \quad \Rightarrow \quad \ddot{\mathbf{r}} \times \mathbf{h} = \frac{\mu}{r^3}\left[\dot{\mathbf{r}}(\mathbf{r}.\mathbf{r}) - \mathbf{r}(\mathbf{r}.\dot{\mathbf{r}})\right]$$

$$\mathbf{r}.\dot{\mathbf{r}} = r\dot{r} \quad \Rightarrow \quad = \mu\left(\frac{\dot{\mathbf{r}}}{r} - \frac{\mathbf{r}\dot{r}}{r^2}\right) = \mu\frac{d}{dt}\left(\frac{\mathbf{r}}{r}\right)$$

$$\int \quad \Rightarrow \quad \boxed{\dot{\mathbf{r}} \times \mathbf{h} - \mu\frac{\mathbf{r}}{r} = \text{constant} = \mu\mathbf{e}}$$

e lies in the orbit plane (***e*.h**)=0: the line defined by **e** is the apse line. Its norm, *e*, is the eccentricity.

Note: demonstrate the Identity $\mathbf{r}.\dot{\mathbf{r}} = r\dot{r}$

$$\frac{d}{dt}(\mathbf{r}.\mathbf{r}) = \mathbf{r}.\frac{d\mathbf{r}}{dt} + \frac{d\mathbf{r}}{dt}.\mathbf{r} = 2\mathbf{r}.\frac{d\mathbf{r}}{dt} = 2\mathbf{r}.\dot{\mathbf{r}}$$

$$\mathbf{r}.\mathbf{r} = r^2 \quad \Rightarrow \quad \frac{d}{dt}(\mathbf{r}.\mathbf{r}) = 2r\frac{dr}{dt} = 2r\dot{r}$$

$$\Downarrow$$

$$\mathbf{r}.\dot{\mathbf{r}} = r\dot{r}$$

1. Orbit Equation

$$\frac{\dot{\mathbf{r}}\times\mathbf{h}}{\mu}=\frac{\mathbf{r}}{r}+\mathbf{e} \quad \xrightarrow{\mathbf{r}.} \quad \frac{\mathbf{r}.\left(\dot{\mathbf{r}}\times\mathbf{h}\right)}{\mu}=\frac{\mathbf{r}.\mathbf{r}}{r}+\mathbf{r}.\mathbf{e}$$

$$\mathbf{a}.\left(\mathbf{b}\times\mathbf{c}\right)=\left(\mathbf{a}\times\mathbf{b}\right).\mathbf{c}$$

$$\frac{\mathbf{r}.\left(\dot{\mathbf{r}}\times\mathbf{h}\right)}{\mu}=\frac{\left(\mathbf{r}\times\dot{\mathbf{r}}\right).\mathbf{h}}{\mu}=\frac{\mathbf{h}.\mathbf{h}}{\mu}=\frac{h^2}{\mu}=r+\mathbf{r}.\mathbf{e}$$

$$\boxed{r=\frac{h^2}{\mu}\frac{1}{1+e\cos\theta}}$$

Closed form of the nonlinear equations of motion (θ is the true anomaly)

1. Energy Conservation (Redundant)

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3}\mathbf{r}$$

$$\ddot{\mathbf{r}}.\dot{\mathbf{r}} = \frac{1}{2}\frac{d}{dt}(\dot{\mathbf{r}}.\dot{\mathbf{r}}) = \frac{1}{2}\frac{d}{dt}\left(\dot{r}^{\mathbf{2}}\right) = \frac{1}{2}\frac{d}{dt}\left(v^{\mathbf{2}}\right)$$

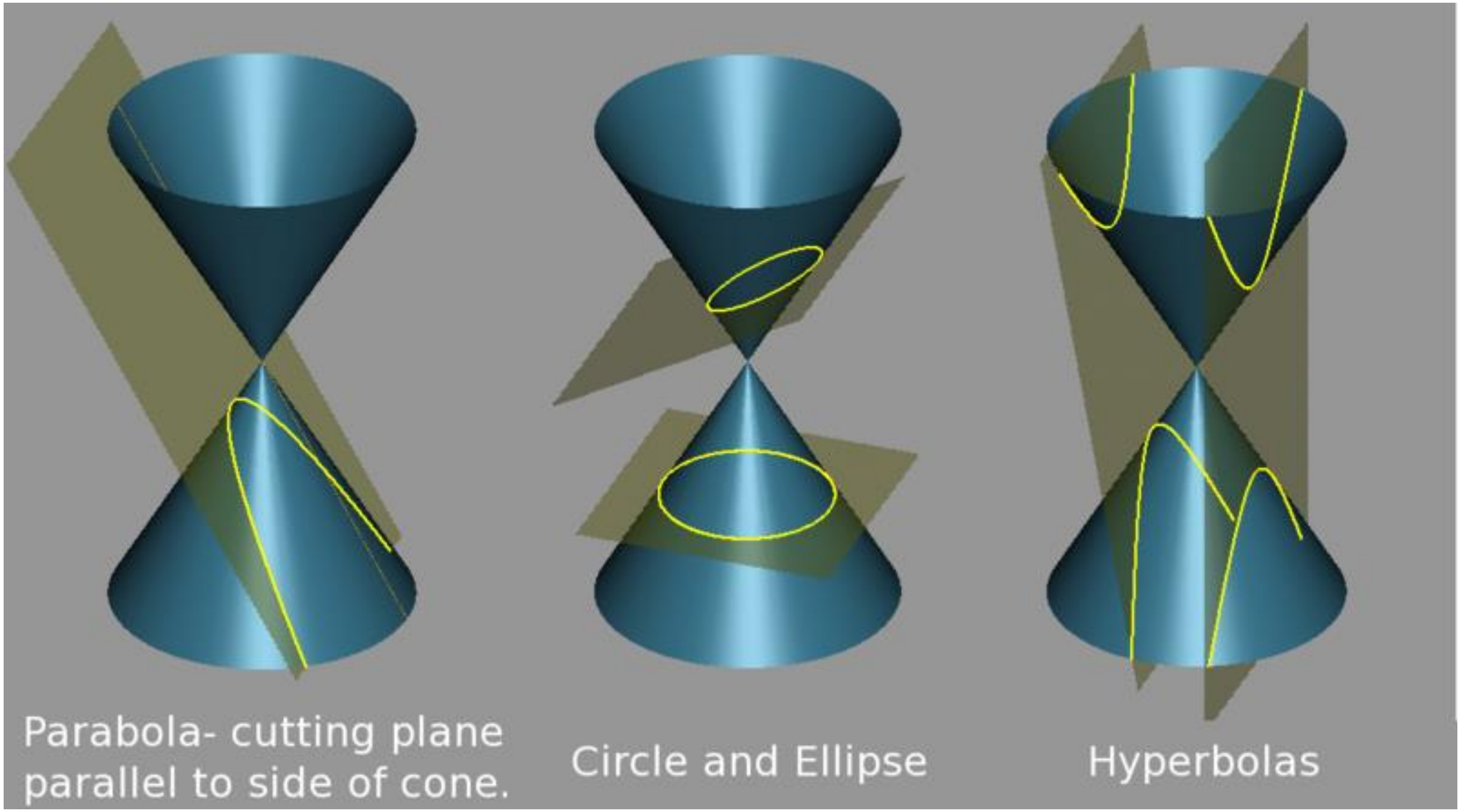
$$\mu\frac{\mathbf{r}.\dot{\mathbf{r}}}{r^3} = \mu\frac{r.\dot{r}}{r^3} = \mu\frac{\dot{r}}{r^2} = -\frac{d}{dt}\left(\frac{\mu}{r}\right)$$

$$\Downarrow$$

$$\frac{v^2}{2} - \frac{\mu}{r} = E$$

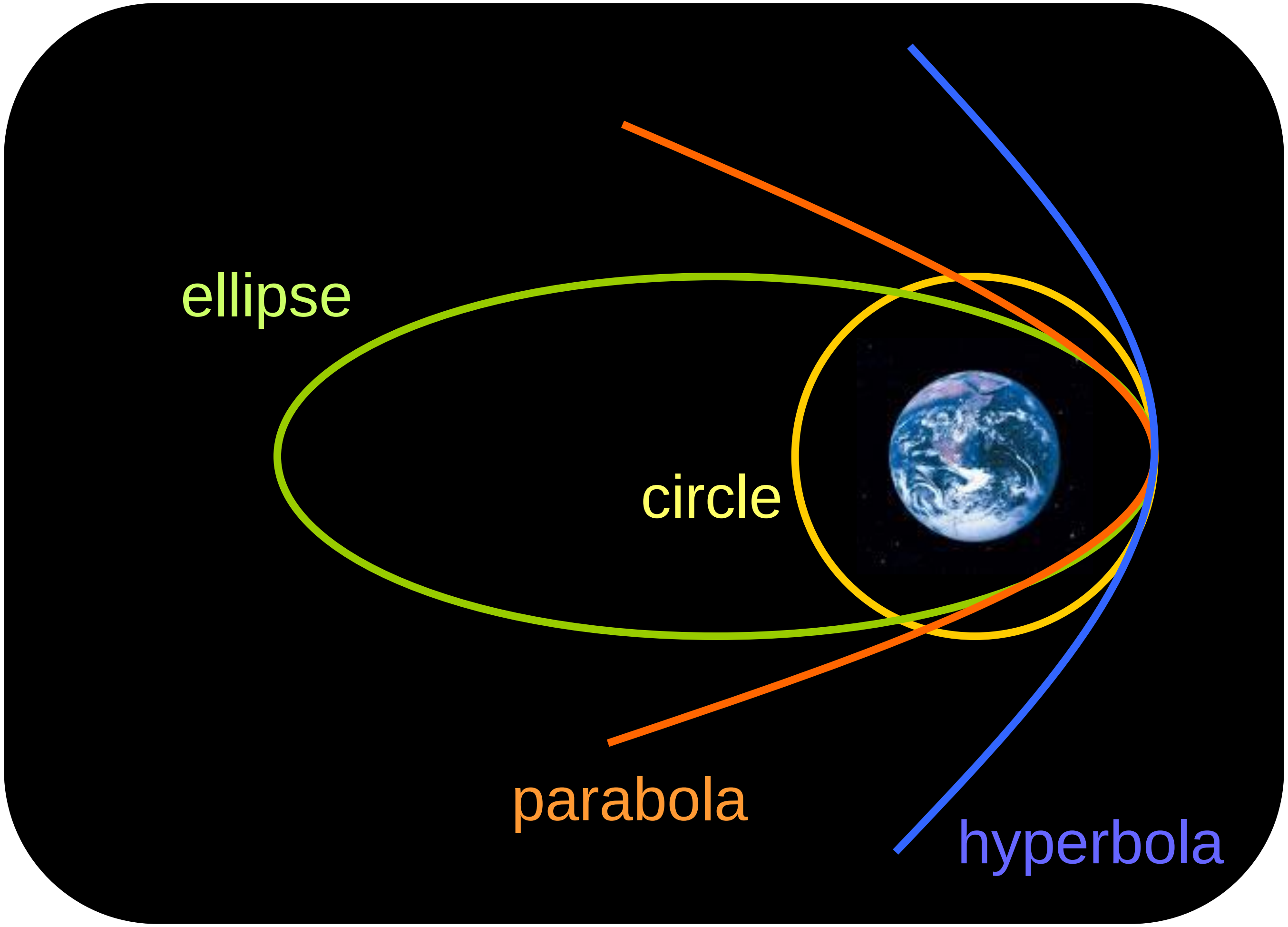
1. Conic Section

$$r = \frac{p}{1 + e\cos\theta}$$



e=1 e=0 0<e<1 e>1

1. Possible Motions in the 2-Body System



1. How Many Variables to Define An Orbit ?

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3}\mathbf{r}$$

6

3 ODEs of second-order

X:	526.375 km
Y:	-5773.15 km
Z:	3409.81 km
X Velocity:	5.76514 km/sec
Y Velocity:	-2.21072 km/sec
Z Velocity:	-4.60703 km/sec

Useful parametrization of the orbit ?

ISS cartesian parameters on March 4, 2009, 12:30:00 UTC (Source: Celestrak)

1. Cartesian Coordinates ?

$\mathbf{r}$ and $\dot{\mathbf{r}}$ do not directly yield much information about the orbit.

We cannot even infer from them what type of conic the orbit represents or what is the orbit altitude !

Another set of six variables, which is much more descriptive of the orbit, is needed.

1. Six Orbital (Keplerian) Elements

1. e: shape of the orbit

2. a: size of the orbit

definition of the ellipse

3. i: orients the orbital plane with respect to the ecliptic plane

4. Ω: longitude of the intersection of the orbital and ecliptic planes

definition of the orbital plane

5. ω: orients the semi-major axis with respect to the ascending node

orientation of the ellipse within the orbital plane

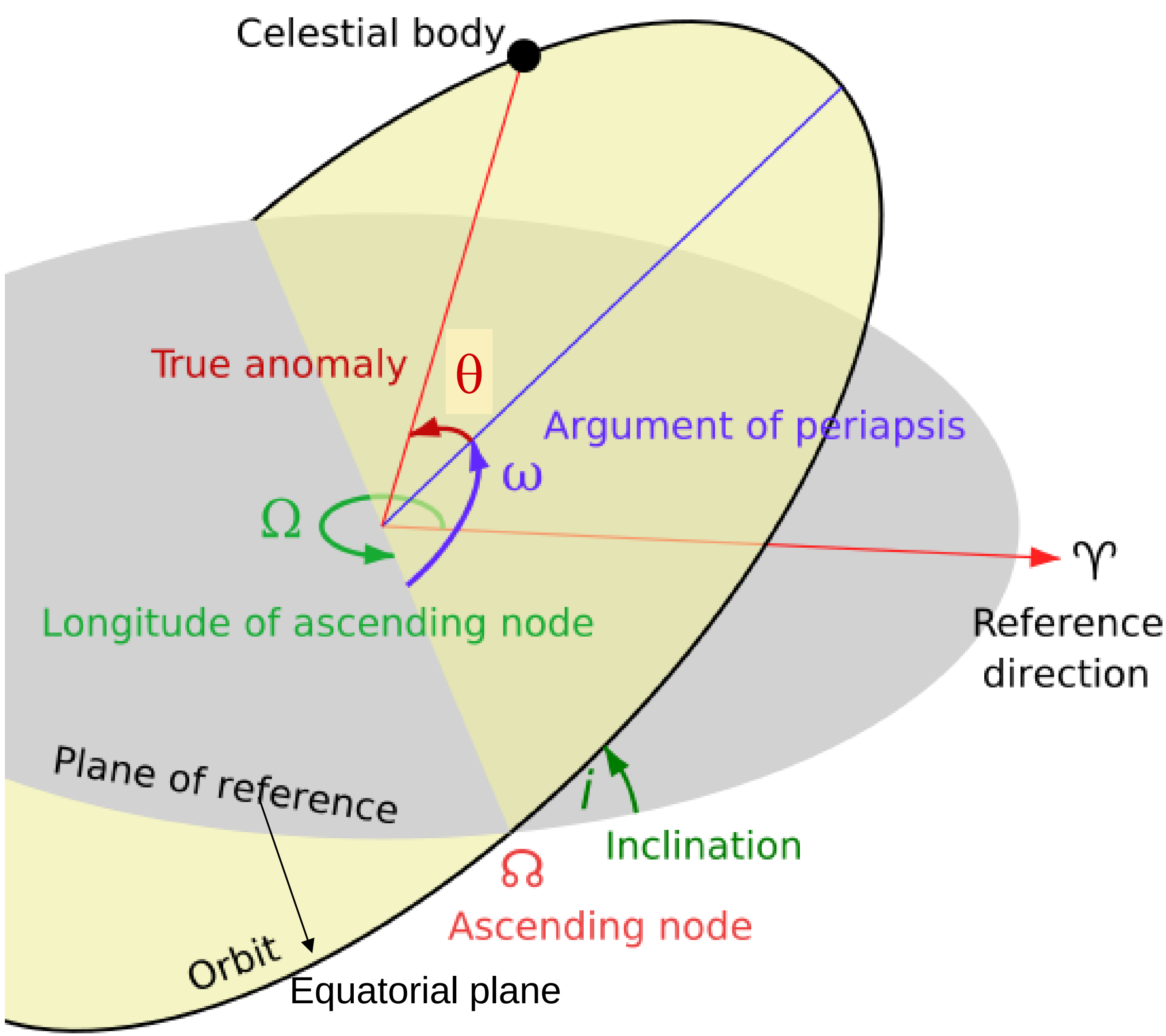
6. θ: orients the celestial body in space

position of the satellite on the ellipse

Orbital plane

orientation of the ellipse

position of the satellite



1. In Summary

+ We can calculate *r* for all values of the true anomaly.

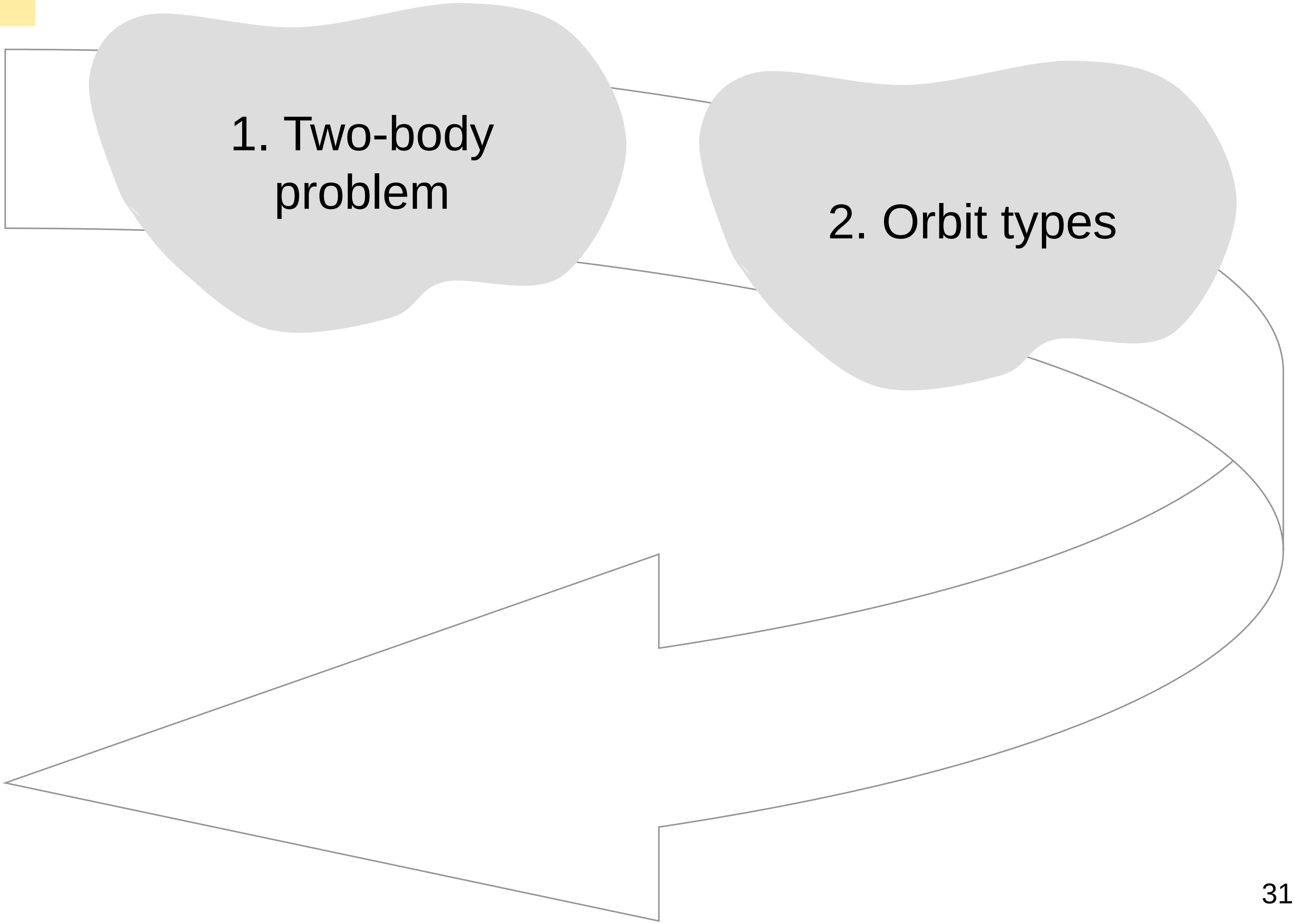
+ The orbit equation is a mathematical statement of Kepler's first law.

- The solution of the "simple" problem of two bodies cannot be expressed in a closed form, explicit function of time.

Do we have 6 independent constants ?

The two vector constants **h** and **e** provide only 5 independent constants: **h.e**=0

Satellite Orbits



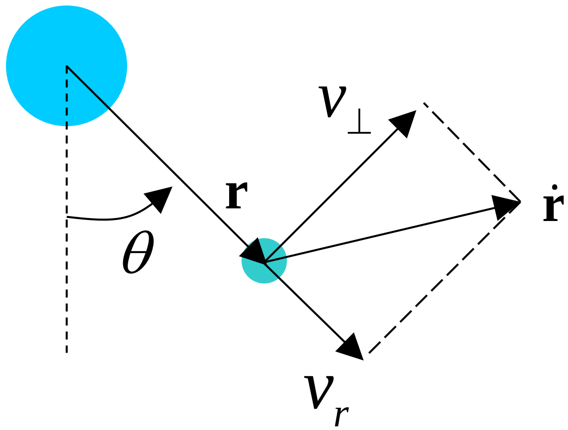
2.1 Circular Orbits (e=0)

$$r = \frac{h^2}{\mu} = \text{Constant} \qquad h = rv_\perp = rv_{circular}$$

$$\Downarrow$$

$$\boxed{v_{circ} = \sqrt{\frac{\mu}{r}}} \qquad \boxed{T_{circ} = 2\pi r \Big/ \sqrt{\frac{\mu}{r}} = \frac{2\pi}{\sqrt{\mu}} r^{3/2}}$$

1. Azimuth Component of the Velocity

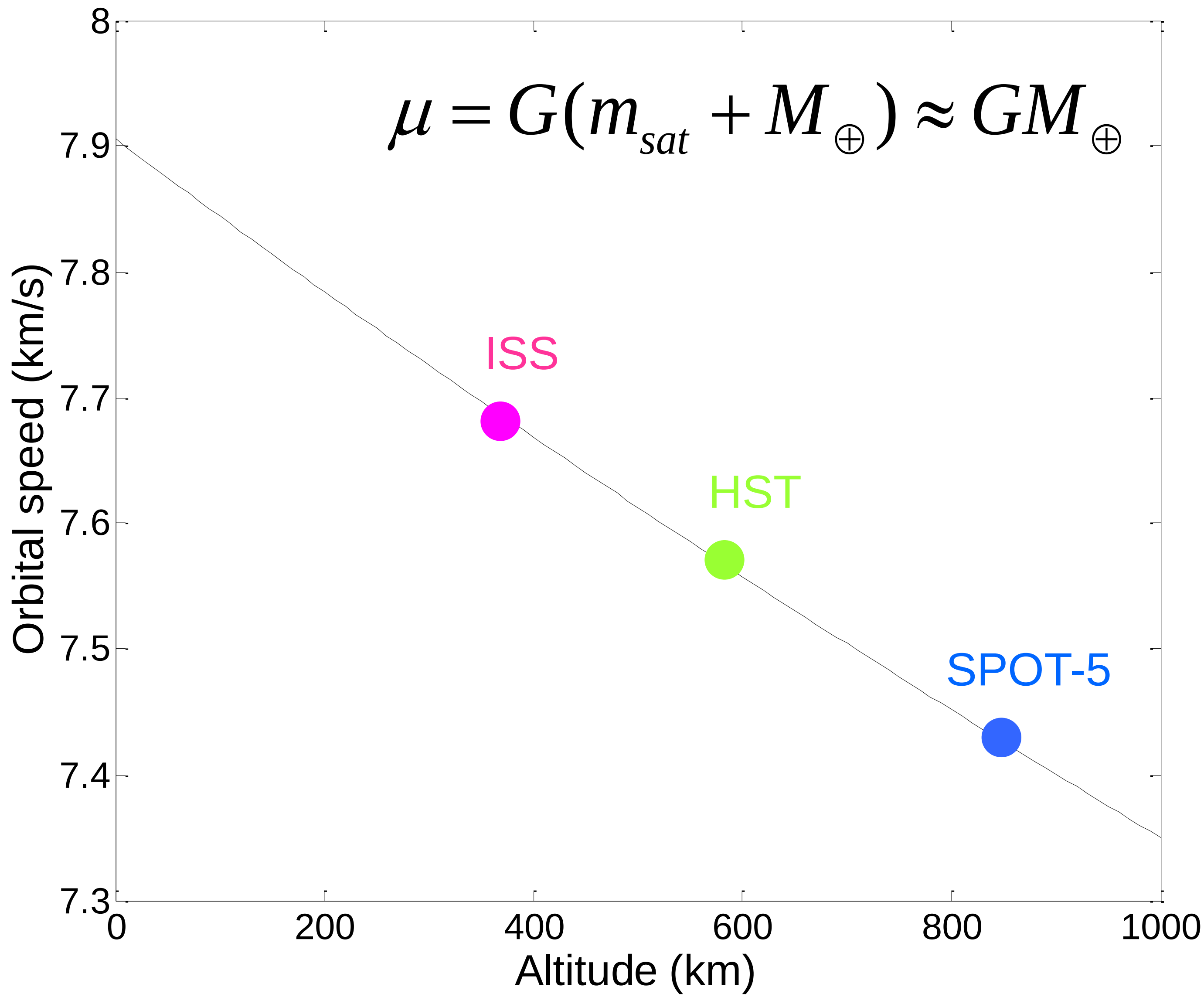


$$\mathbf{h} = \mathbf{r} \times \dot{\mathbf{r}} = r\hat{\mathbf{u}}_r \times (v_r\hat{\mathbf{u}}_r + v_\perp\hat{\mathbf{u}}_\perp) = rv_\perp\hat{\mathbf{h}}$$

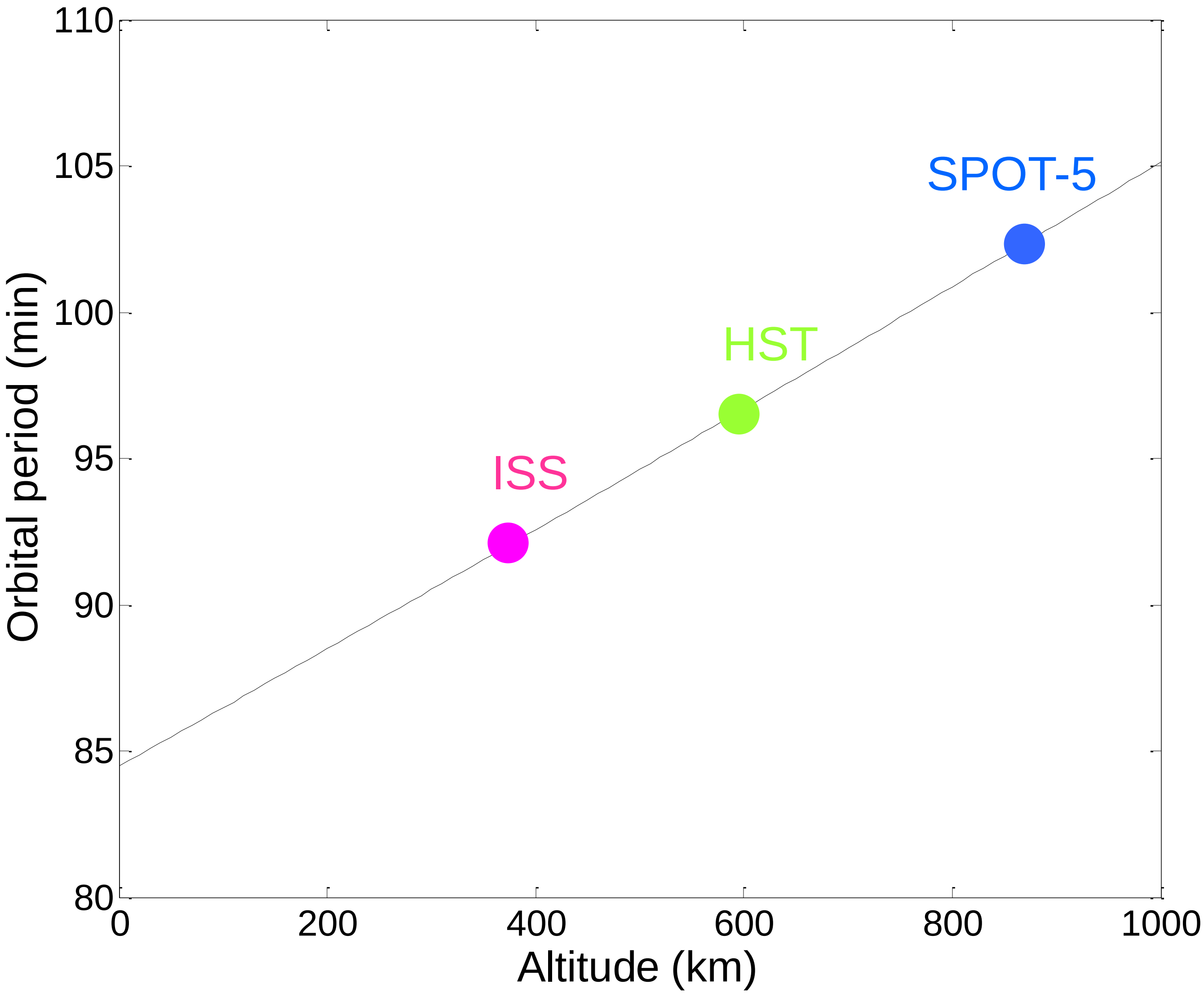
$$\boxed{h = rv_\perp = r^2\dot{\theta}}$$

The angular momentum depends only on the azimuth component of the relative velocity

2.1 Orbital Speed Decreases with Altitude



2.1 Orbital Period Increases With Altitude



2.1 Two Important Cases

1. 7.9 km/s is the **first cosmic velocity**; i.e., the minimum velocity (theoretical velocity, $r = 6378$ km) to orbit the Earth.

2. 35786 km is the altitude of the **geostationary orbit**. It is the orbit at which the satellite angular velocity is equal to that of the Earth, $\omega = \omega_E = 7.292\ 10^{-5}$ rad/s, in inertial space (*).

$$r_{GEO} = \left(\frac{T_{circ}\sqrt{\mu}}{2\pi} \right)^{2/3}$$

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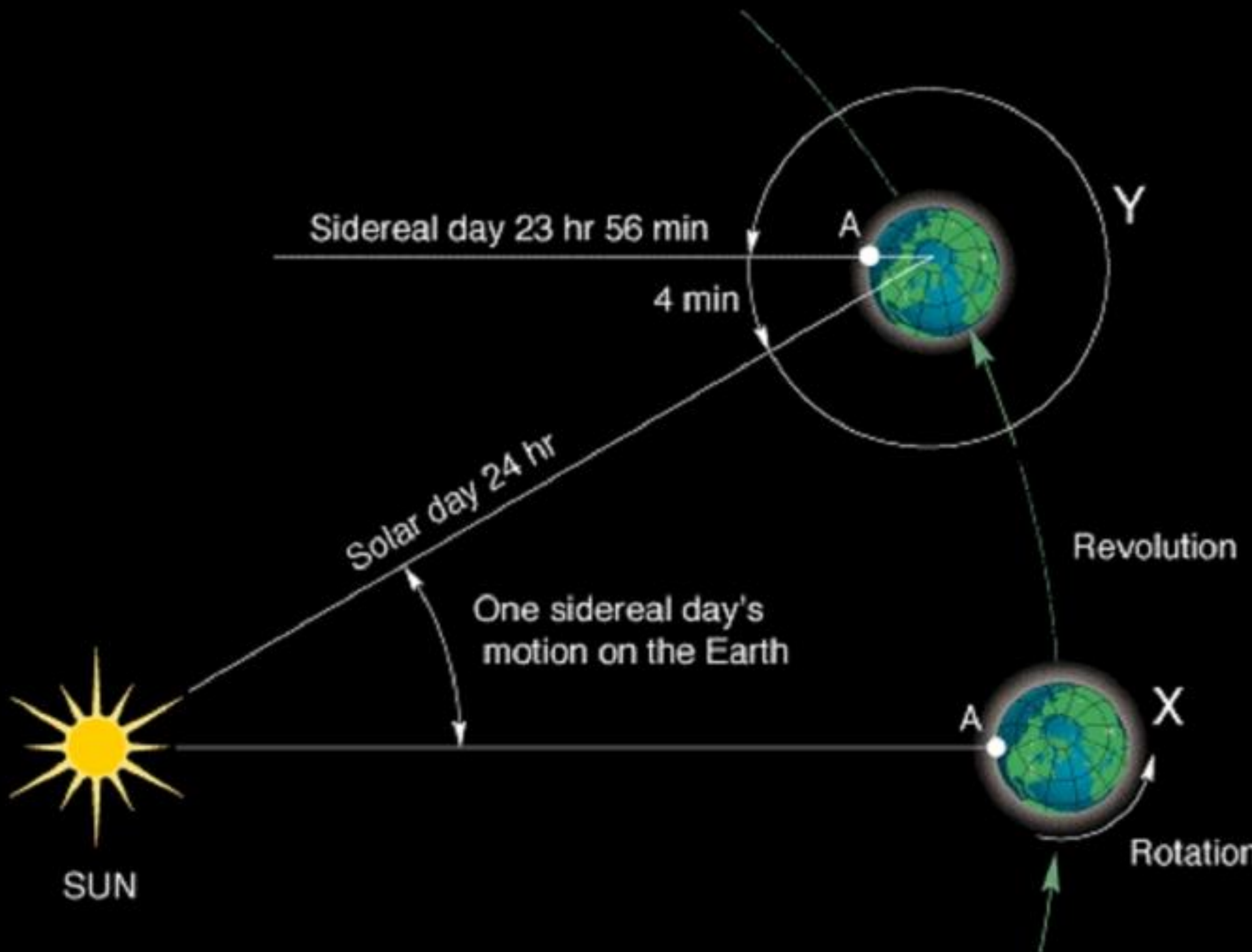
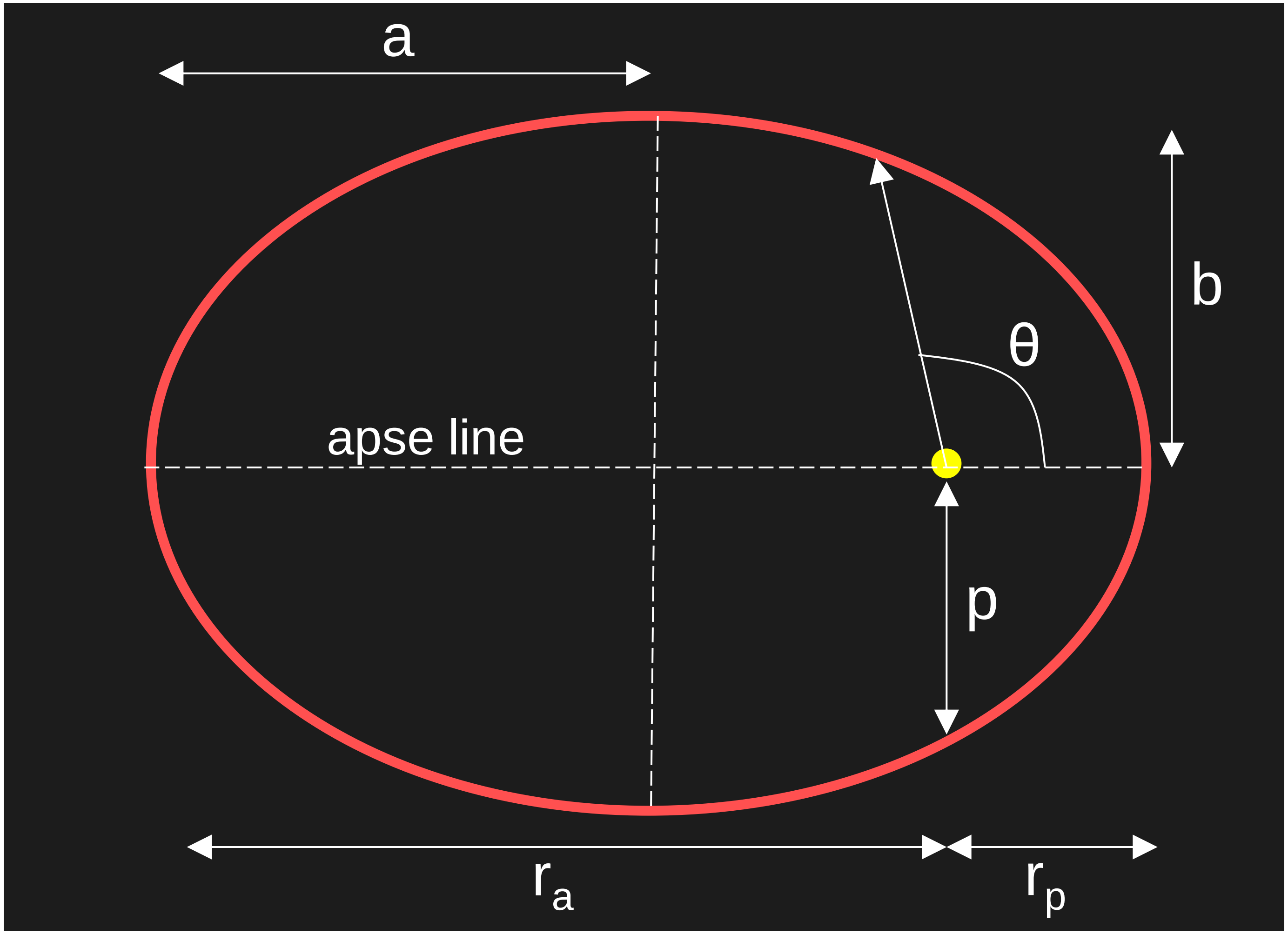
* A sidereal day, 23h56m4s, is the time it takes the Earth to complete one rotation relative to inertial space. A synodic day, 24h, is the time it takes the sun to apparently rotate once around the Earth. They would be identical if the earth stood still in space.

Sidereal Day vs. Solar Day



2.2 Geometry of the Elliptic Orbit



2.2 Elliptic Orbits (0<e<1)

$$r = \frac{h^2}{\mu} \frac{1}{1 + e\cos\theta}$$

The relative position vector remains bounded.

θ=0, minimum separation, **periapse**

$$r_p = \frac{h^2}{\mu(1+e)}$$

θ=π, greatest separation, **apoapse**

$$r_a = \frac{h^2}{\mu(1-e)}$$

$$e = \frac{r_a - r_p}{r_a + r_p}$$

2.2 Velocity in an Elliptical Orbit

$$\frac{v^2}{2} - \frac{\mu}{r} = E \qquad\qquad -\frac{\mu}{2a} = E_{perigee}$$

Energy conservation

See next slide

$$\frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$$

$$v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

2.2 Energy of an Elliptical Orbit

$$\frac{v^2}{2} - \frac{\mu}{r} = E \qquad \frac{v_p^2}{2} - \frac{\mu}{r_p} = E_{perigee}$$

$h = v_p r_p$ See part 1

$$\frac{h^2}{2r_p^2} - \frac{\mu}{r_p} = E_{perigee}$$

$$r_p = \frac{h^2}{\mu(1+e)}$$

$$-\frac{1}{2}\frac{\mu^2}{h^2}(1 - e^2) = E_{perigee}$$

Link between energy and the other constants **h** and **e**!

$h = \sqrt{\mu a(1-e^2)}$ See next slide

$$-\frac{\mu}{2a} = E_{perigee}$$

2.2 Angular Momentum

$$r = \frac{h^2}{\mu}\frac{1}{1+e\cos\theta}$$

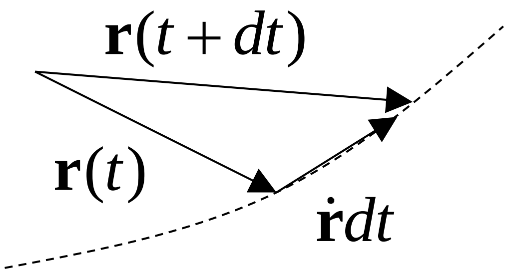
Orbit equation

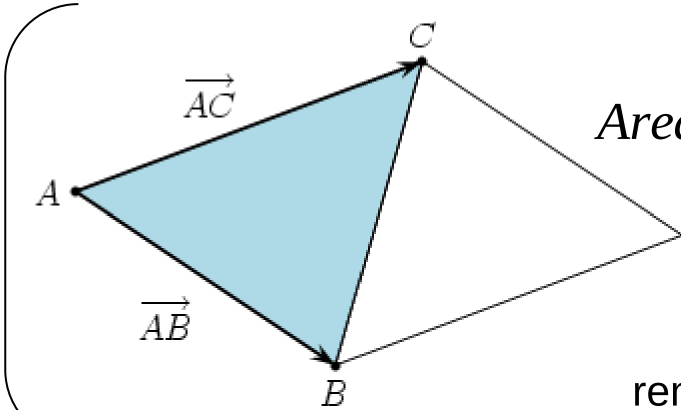
$$r = \frac{a(1-e^2)}{1+e\cos\theta}$$

Polar equation of an ellipse
(*a*, semimajor axis)

$$\boxed{h = \sqrt{\mu a(1-e^2)}}$$

2.2 Kepler's Second Law





$$Area = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$$

reminder

$$dA = \frac{1}{2}\left|\mathbf{r} \times \dot{\mathbf{r}} dt\right| = \frac{1}{2}\left|\mathbf{h}\right| dt = \frac{1}{2} h dt$$

$$\boxed{\frac{dA}{dt} = \frac{h}{2} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \text{constant}}$$

The line from the sun to a planet sweeps out equal areas inside the ellipse in equal lengths of time.

2.2 Kepler's Third Law

$$T = \frac{\text{enclosed area}}{dA/dt} = \frac{2\pi \text{ab}}{h}$$

$$h = \sqrt{\mu a(1-e^2)} \qquad b = a\sqrt{1-e^2}$$

$$\boxed{T_{ellip} = 2\pi\sqrt{\frac{a^3}{\mu}}}$$

The elliptic orbit period depends only on the semimajor axis and is independent of the eccentrivity.

$$\boxed{\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3}}$$

The squares of the orbital periods of the planets are proportional to the cubes of their mean distances from the sun.

2.2 Example (1447km x 354km)

$$r_p = 354 + 6378 = 6732\,\text{km} \qquad r_a = 1447 + 6378 = 7825\,\text{km}$$

$$\begin{cases} e = \dfrac{r_a - r_p}{r_a + r_p} = 0.075, \quad a = \dfrac{r_a + r_p}{2} = 7278.5\,\text{km} \\ T = 2\pi\sqrt{\dfrac{a^3}{\mu}} = 6179.79\text{s} = 103\,\text{min} \\ v = \sqrt{\mu\left(\dfrac{2}{r} - \dfrac{1}{a}\right)} \end{cases}$$

$$v_p = 7.98\,\text{km/s}$$

$$v_a = 6.86\,\text{km/s}$$

2.3 Parabolic Orbits (e=1)

$$r = \frac{h^2}{\mu} \frac{1}{1+\cos\theta} \qquad \theta \to \pi,\ r \to \infty$$

$$v_{parab} = \sqrt{\frac{2\mu}{r}}$$

The satellite will coast to infinity, arriving there with zero velocity relative to the central body.

2.3 Escape Velocity, V_{esc}

11.2 km/s is the **second cosmic velocity**; i.e., the minimum velocity (theoretical velocity, $r = 6378$km) to escape the gravitational attraction of the Earth.

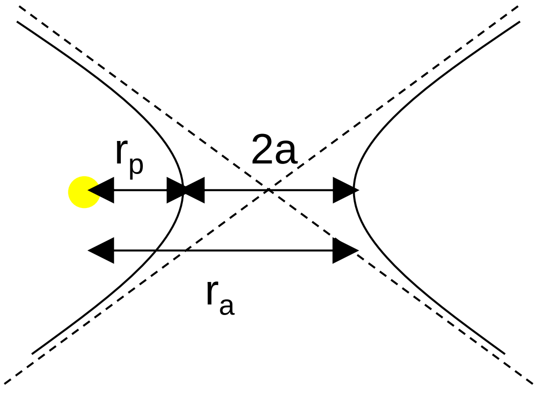
$$v_{circ} = \sqrt{\frac{\mu}{r}} \qquad v_{parab} = \sqrt{\frac{2\mu}{r}}$$

$$\Downarrow$$

$$11.2\,\text{km/s} = \sqrt{2} \times 7.9\,\text{km/s}$$

2.4 Hyperbolic Orbits (e>1)

$$r = \frac{h^2}{\mu}\frac{1}{1+e\cos\theta}$$



$$v_\infty = \sqrt{\frac{\mu}{a}}$$

Hyperbolic excess speed

$$v^2 = v_\infty^2 + v_{esc}^2 = C_3 + v_{esc}^2$$

C_3 is a measure of the energy for an interplanetary mission:

16.6 km^2/s^2 (Cassini-Huygens)

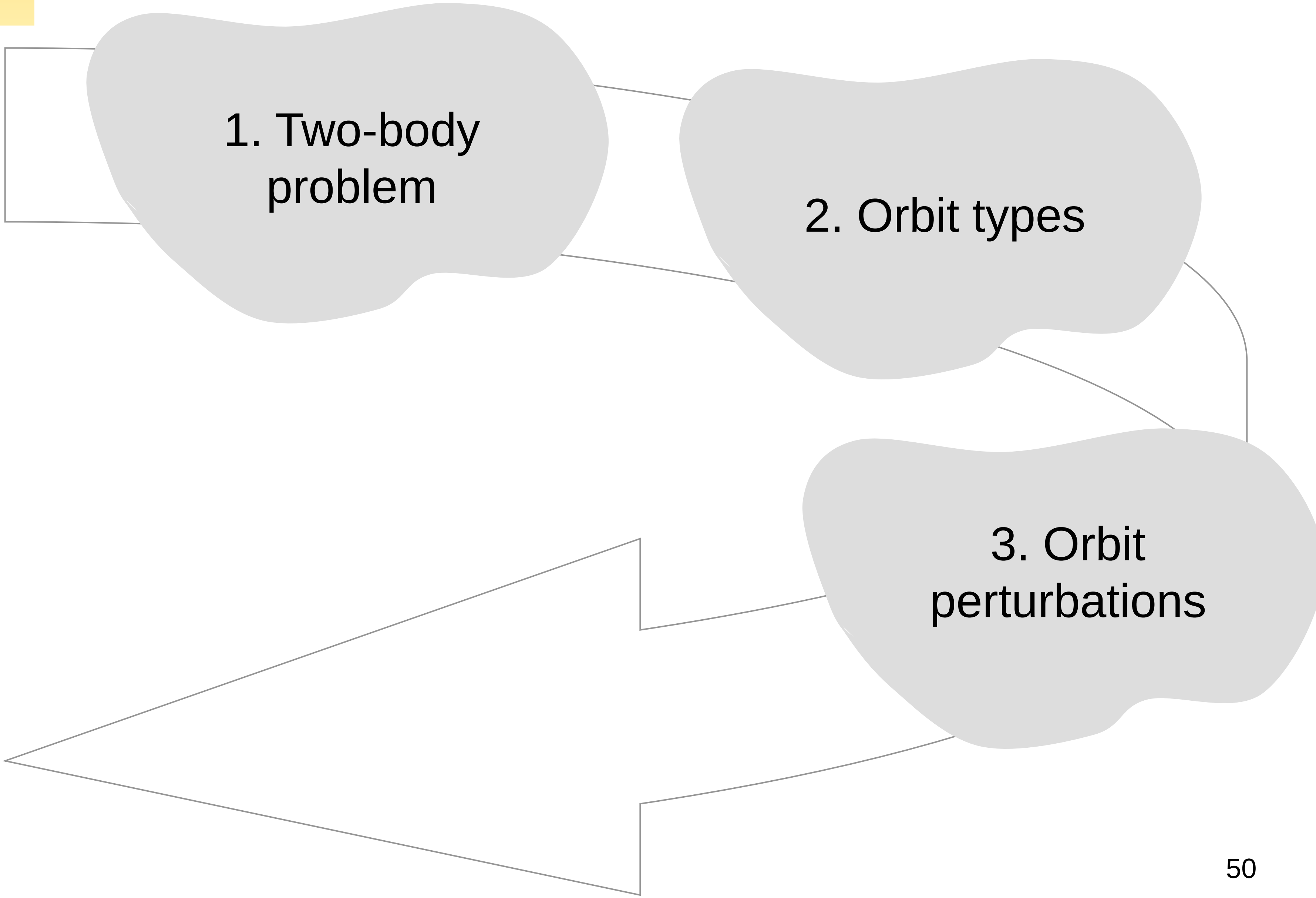
8.9 km^2/s^2 (Solar Orbiter, phase A)

What Do you Think ?

Assume we have a circular or elliptic orbit for our satellite.

Will it stay there ???

Satellite Orbits



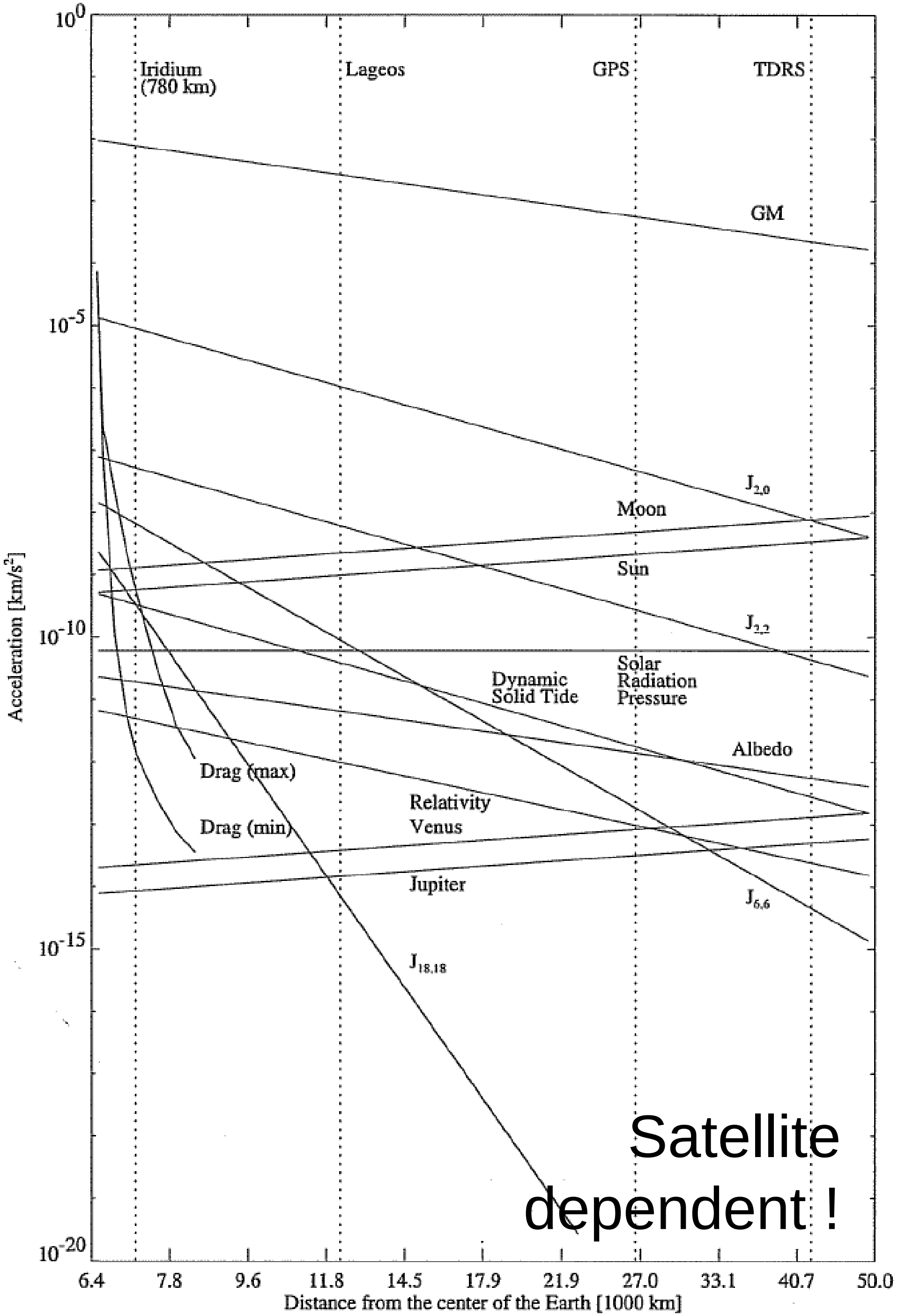
3.1 Non-Keplerian Motion

In many practical situations, a satellite experiences significant perturbations (accelerations).

These perturbations are sufficient to cause predictions of the position of the satellite based on a Keplerian approach to be in significant error in a brief time.

Different Perturbations ?

LEO ? GEO ?

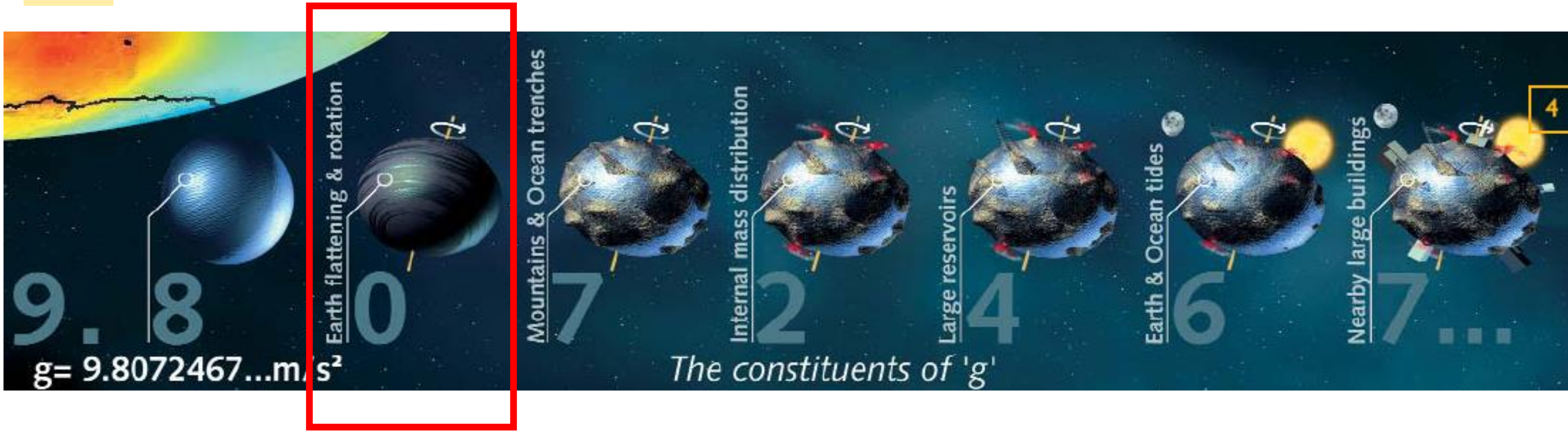


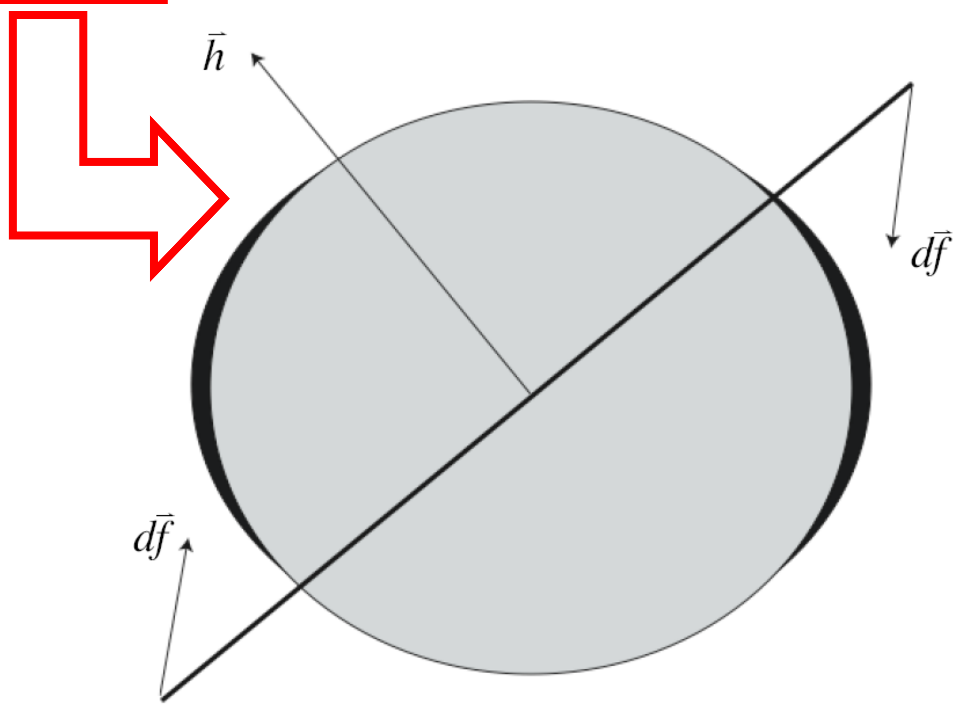
Montenbruck and Gill, *Satellite orbits,* Springer, 2000

3.1 Respective Importance

400 kms	1000 kms	36000 kms
Oblateness	**Oblateness**	Oblateness
Drag	Sun and moon	Sun and moon
		SRP

3.2 The Earth is not a Sphere…



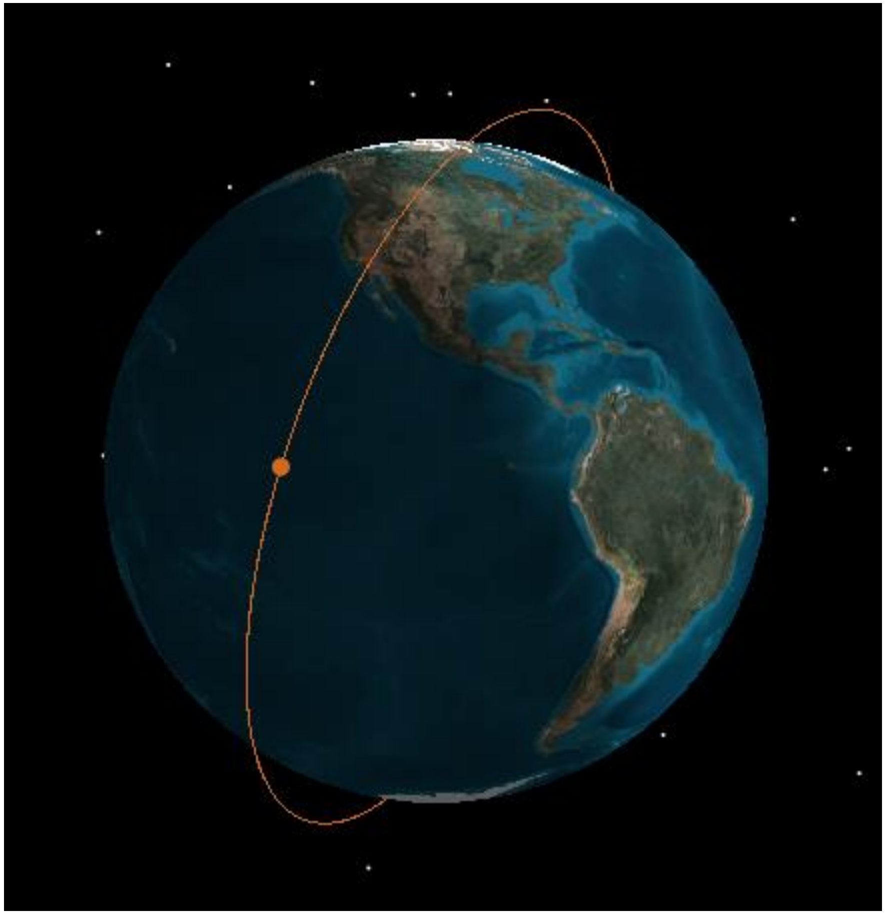


$$\ddot{\vec{r}} = -\frac{\mu \vec{r}}{r^3} + d\vec{f}$$

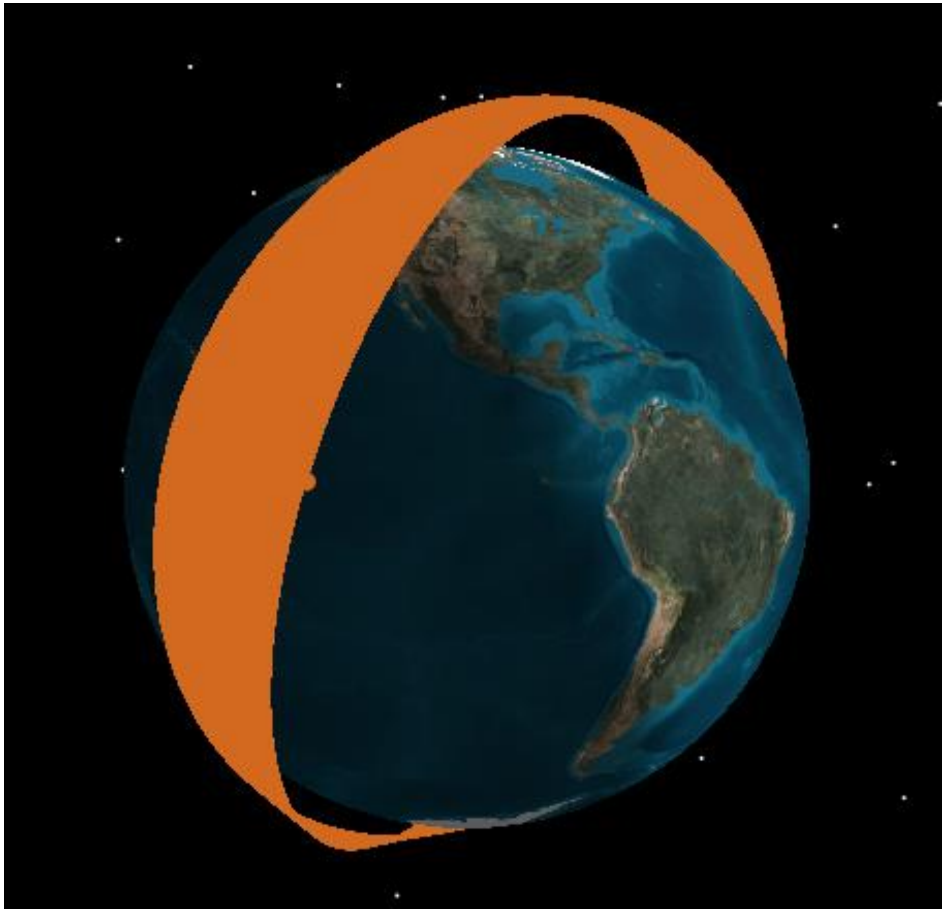
$$\vec{r} \times \ddot{\vec{r}} = -\vec{r} \times \frac{\mu \vec{r}}{r^3} + \vec{r} \times d\vec{f}$$

$$\frac{d}{dt}\left(\vec{r} \times \dot{\vec{r}}\right) = \dot{\vec{h}} = \vec{r} \times d\vec{f}$$

3.2 The Earth is not a Sphere



Two-body propagator



J2 propagator

3.2 Physical Interpretation

The force of gravity is no longer within the orbital plane: **non-planar motion will result**.

The equatorial bulge exerts a force that pulls the satellite back to the equatorial plane and thus tries to align the orbital plane with the equator.

Due to its angular momentum, the orbit behaves like a spinning top and reacts with a precessional motion of the orbital plane (the orbital plane of the satellite rotates in inertial space).

3.3 The Earth Has an Atmosphere

Atmospheric forces represent the largest nonconservative perturbations acting on low-altitude satellites.

The drag is directly opposite to the velocity of the satellite.

The lift force can be neglected in most cases.

3.3 Effects of Atmospheric Drag

Start Time:	21 Sep 2009 10:00:00.000 UTC
Stop Time:	1 Mar 2010 14:45:53.000 UTCG

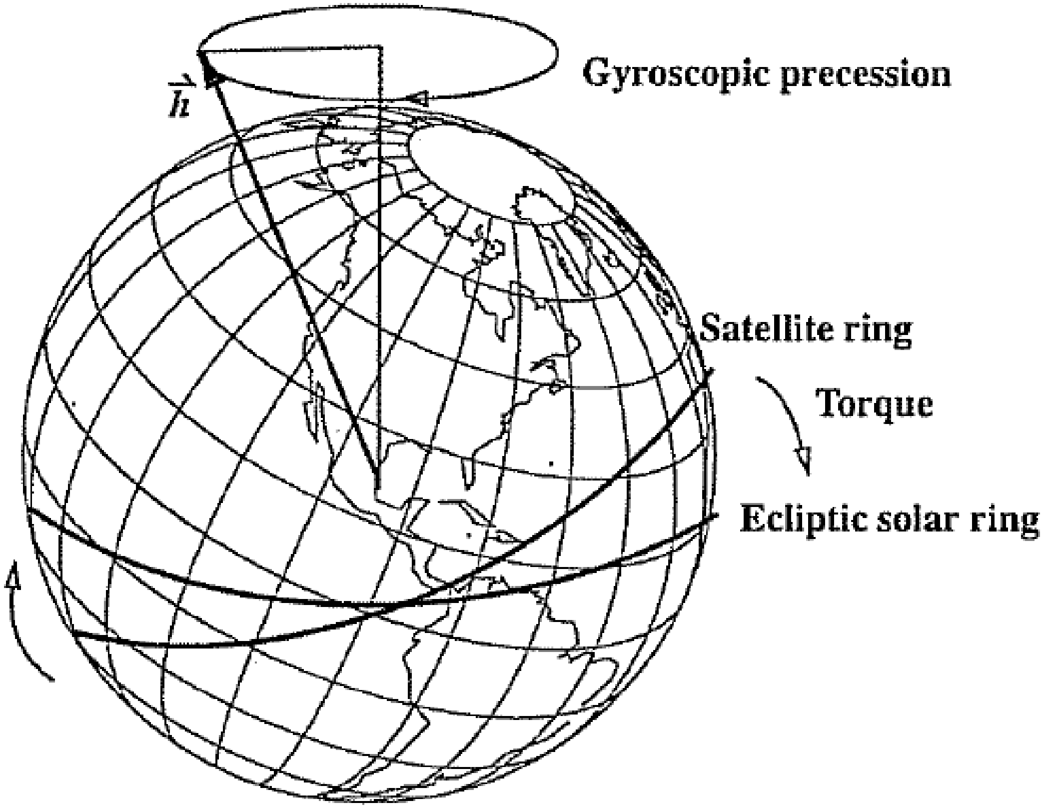
Apogee Altitude	2000 km
Perigee Altitude	250 km
Inclination	71 deg
Argument of Perigee	0 deg
RAAN	180 deg
True Anomaly	0 deg

3.4 Third-Body Perturbations

For an Earth-orbiting satellite, the Sun and the Moon should be modeled for accurate predictions.

Their effects become noticeable when the effects of drag begin to diminish.

3.4 Effects of Third-Body Perturbations



The Sun's attraction tends to turn the satellite ring into the ecliptic. The orbit precesses about the pole of the ecliptic.

Third-Body Interactions. Imagine that the entire mass of a third body (the Sun, for instance) occupies a band about the planet. The resulting torque causes the satellite's orbit to precess like a gyroscope.

Vallado, *Fundamental of Astrodynamics and Applications*, Kluwer, 2001.

3.5 Solar Radiation Pressure

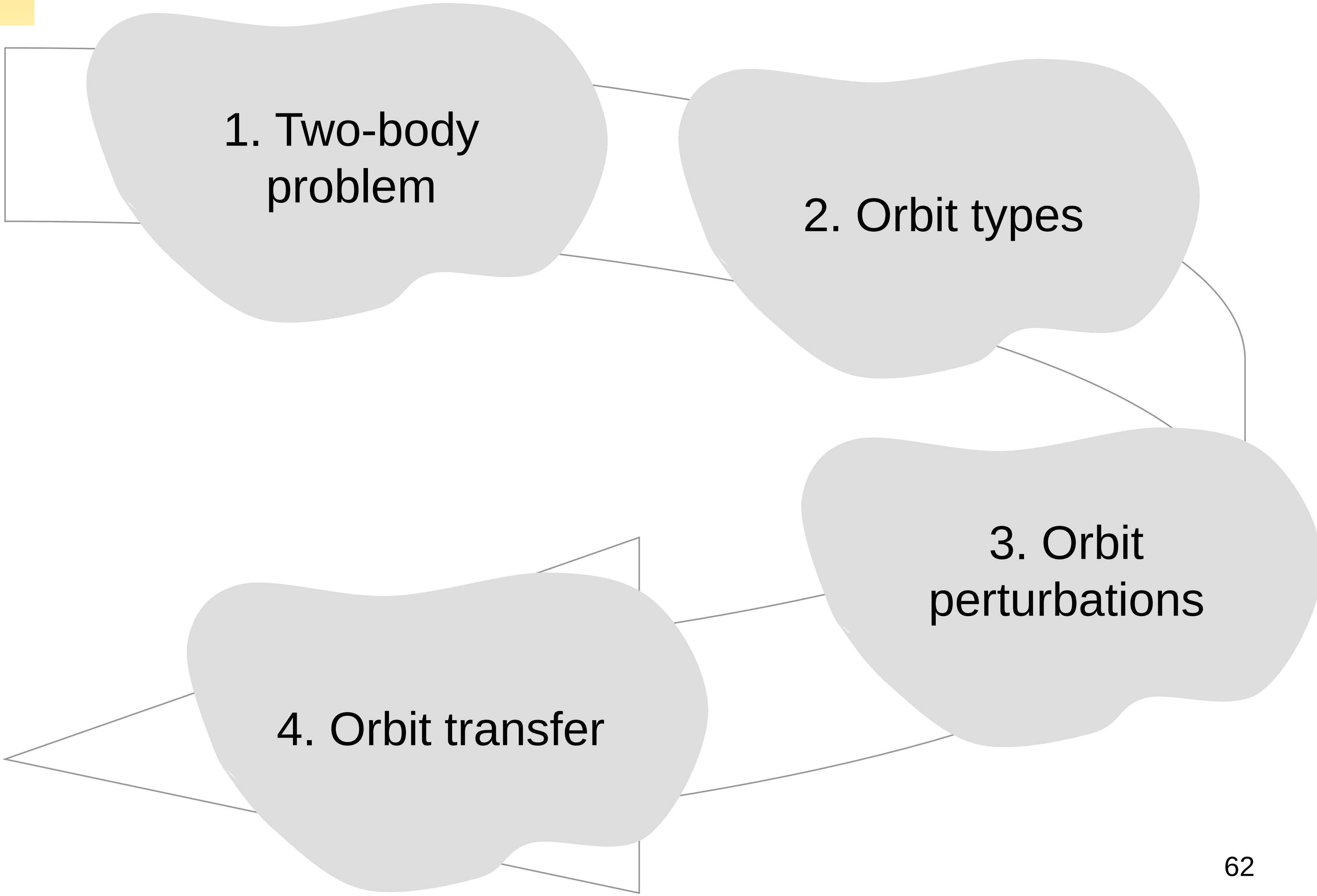
Solar radiation (photons)	$\neq$	Solar wind (particles)

It produces a nonconservative perturbation on the spacecraft, which depends upon the distance from the sun.

It is usually very difficult to determine precisely, but the effects are usually small for most satellites.

800km is regarded as a transition altitude between drag and SRP.

Satellite Orbits



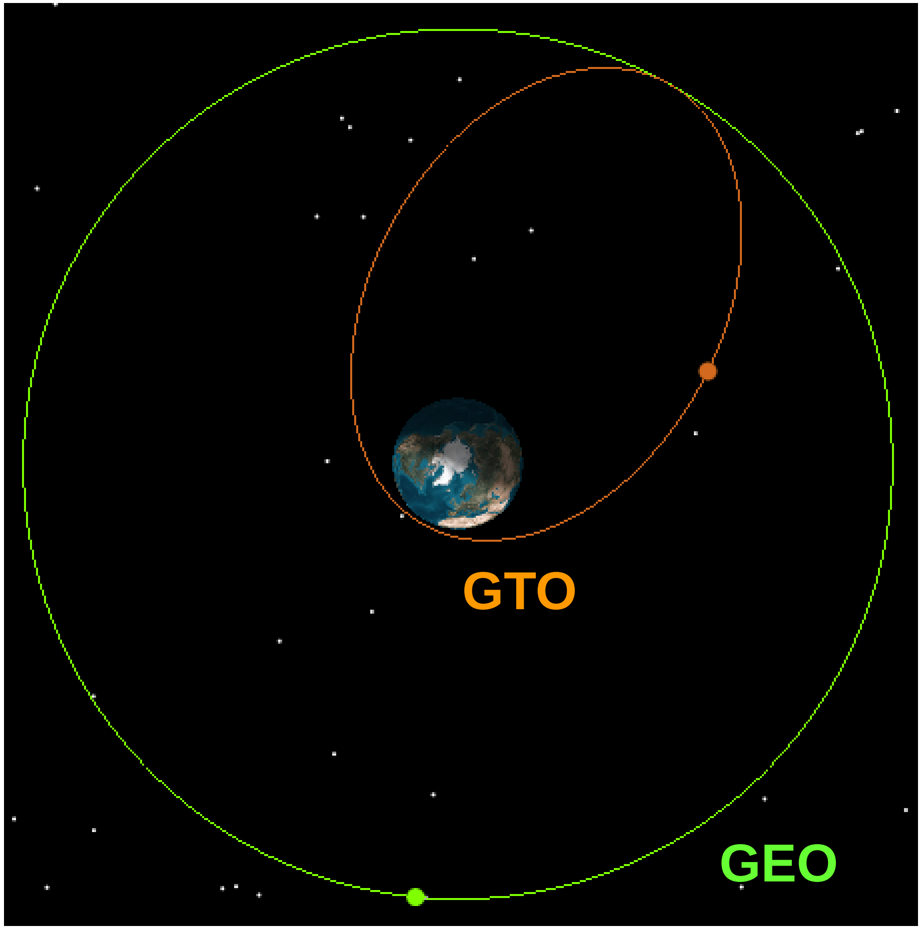
4. Motivation

Without maneuvers, satellites could not go beyond the close vicinity of Earth.

For instance, a GEO spacecraft is usually placed on a transfer orbit (LEO or GTO).

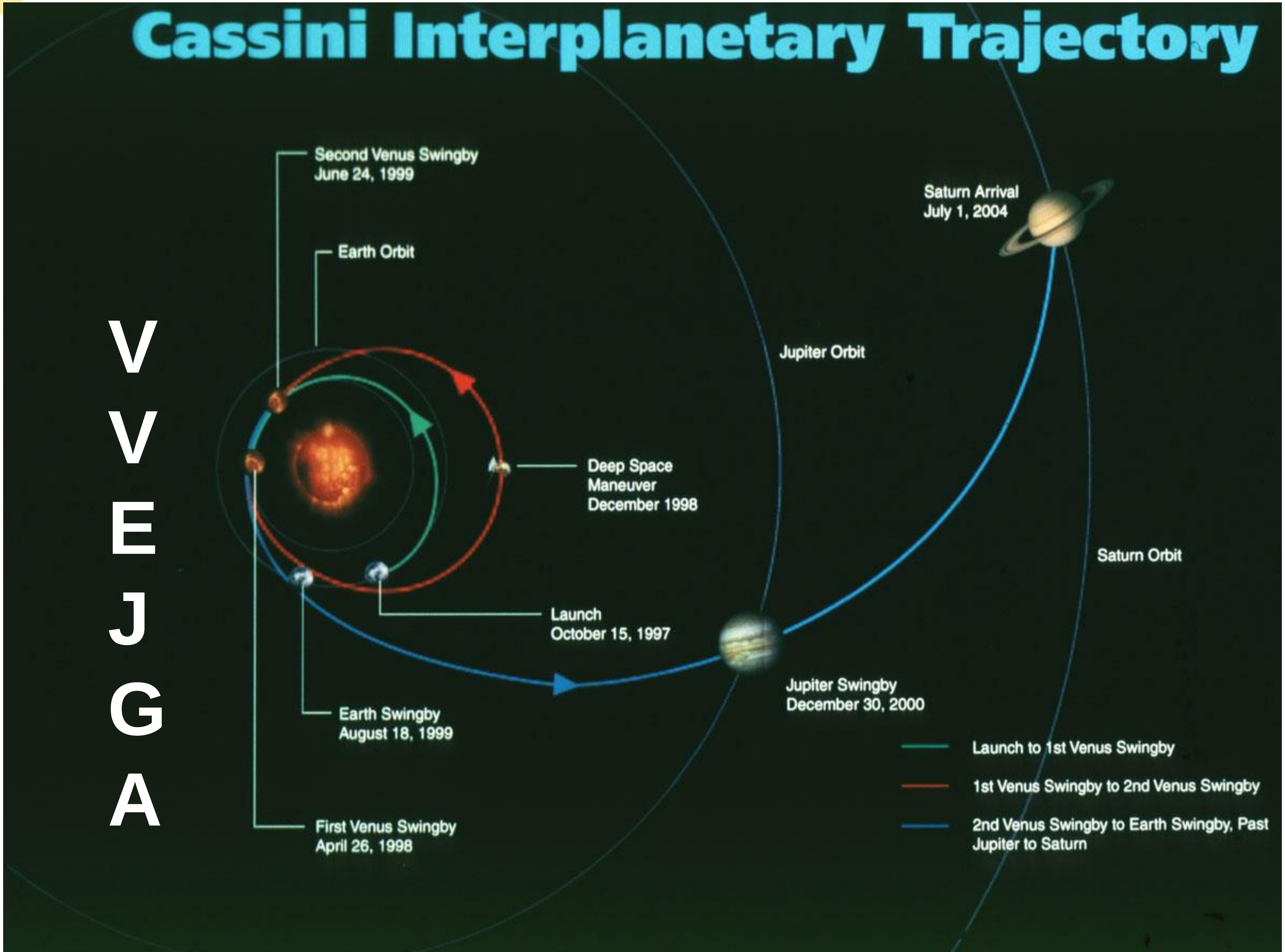
4. From GTO to GEO: Ariane V

Ariane V is able to place heavy GEO satellites in GTO: perigee: 200-650 km and apogee: ~35786 km.



4. How to Go to Saturn ?

V
V
E
J
G
A



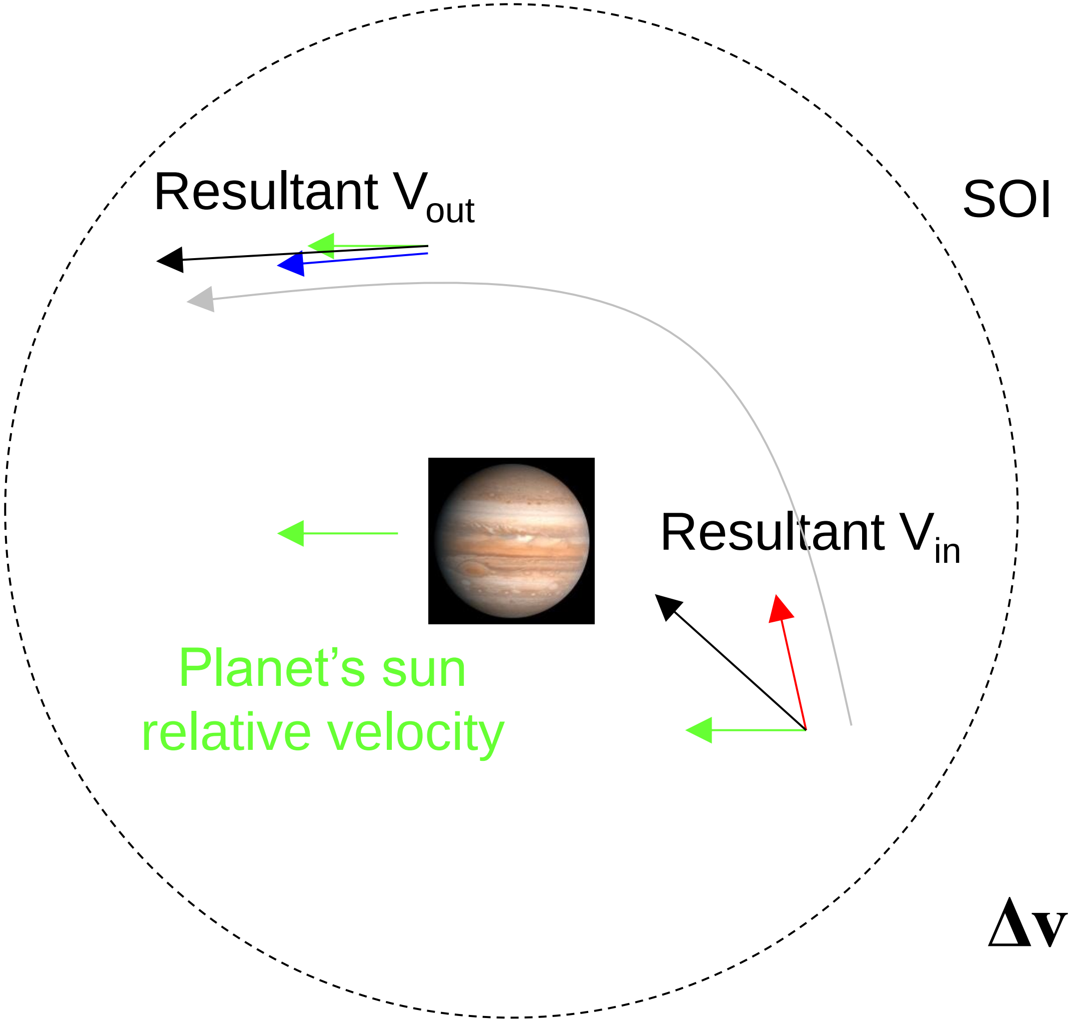
4. Gravity Assist

Also known as planetary flyby trajectory, slingshot maneuver and swingby trajectory.

Useful in interplanetary missions to obtain a velocity change without expending propellant.

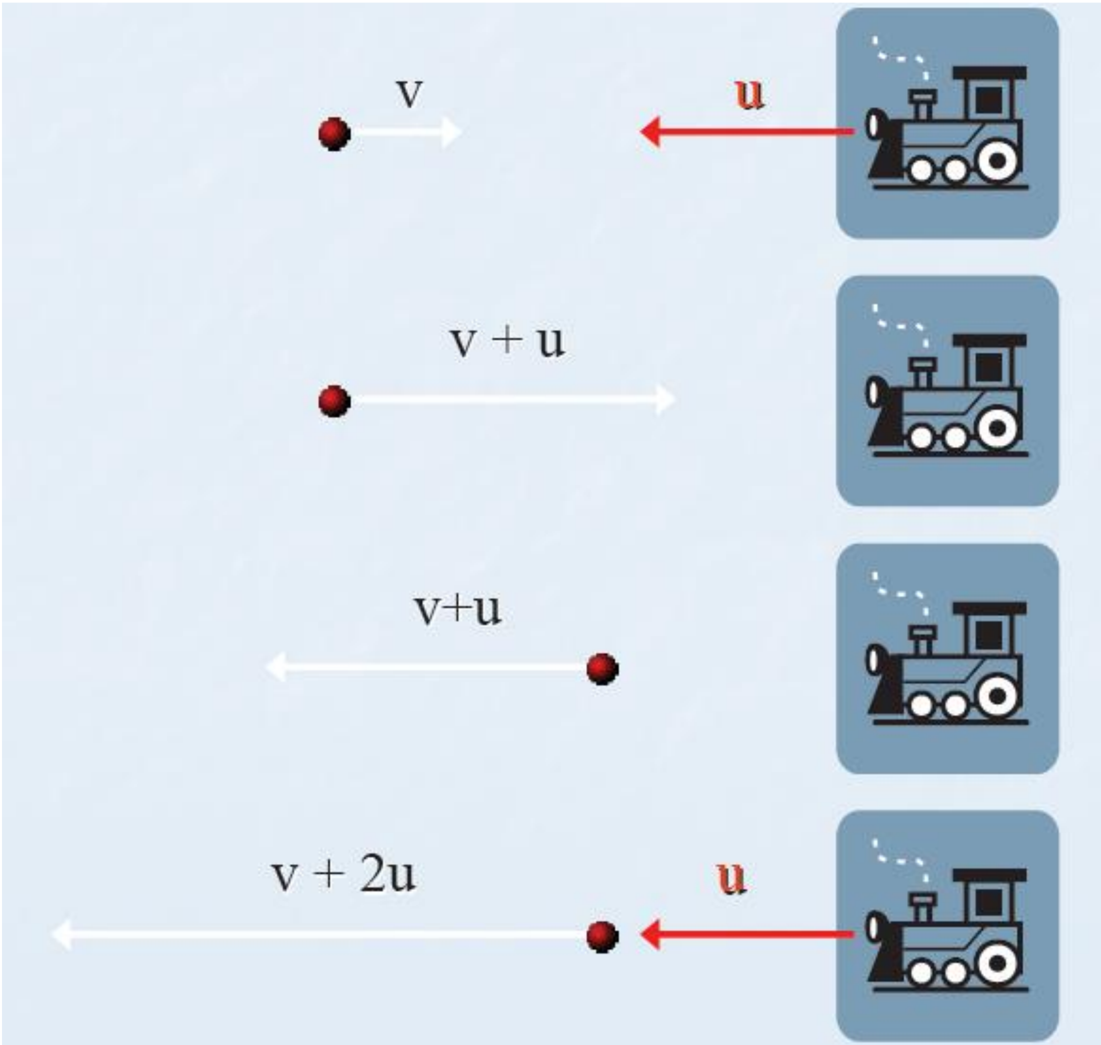
This free velocity change is provided by the gravitational field of the flyby planet and can be used to lower the delta-v cost of a mission.

4. Basic Principle



$$\Delta \mathbf{v} = \mathbf{v}_{\infty,out} - \mathbf{v}_{\infty,in}$$

4. Basic Principle



Inertial frame

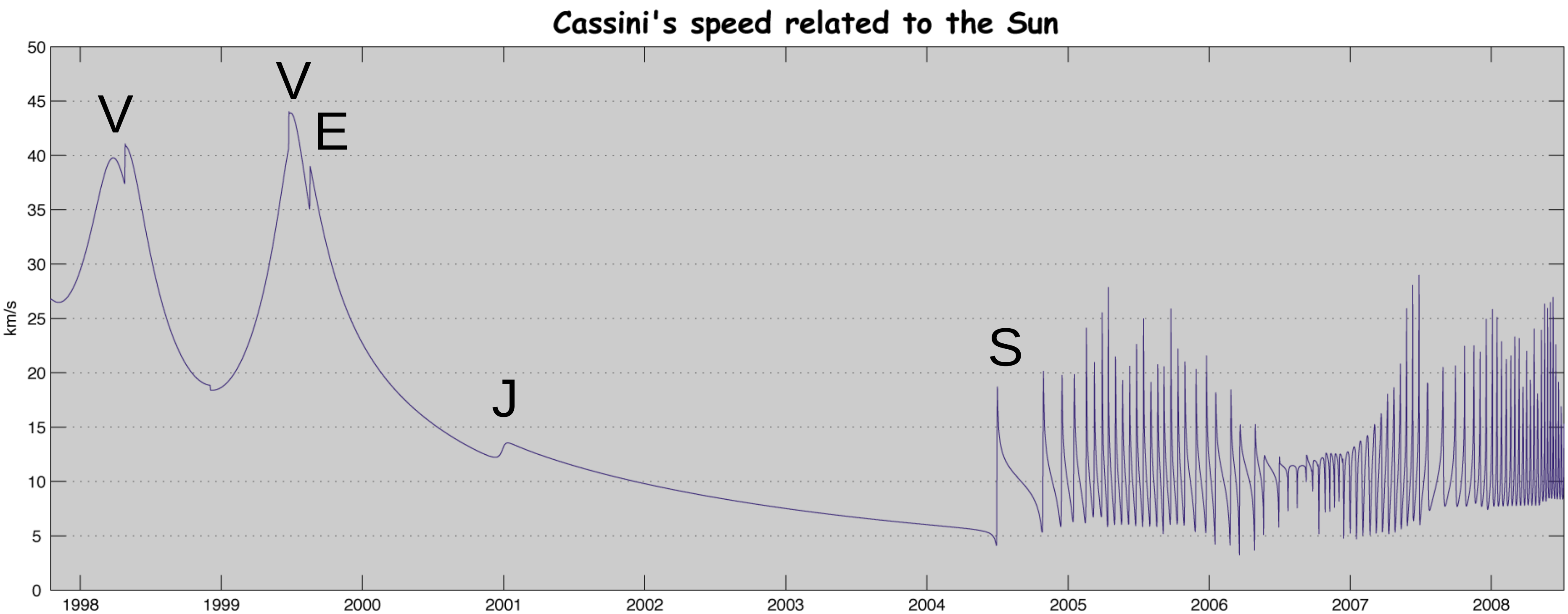
Frame attached to the train

Frame attached to the train

Inertial frame

A gravity assists looks like an elastic collision, although there is no physical contact with the planet.

4. Cassini: Swingby Effects



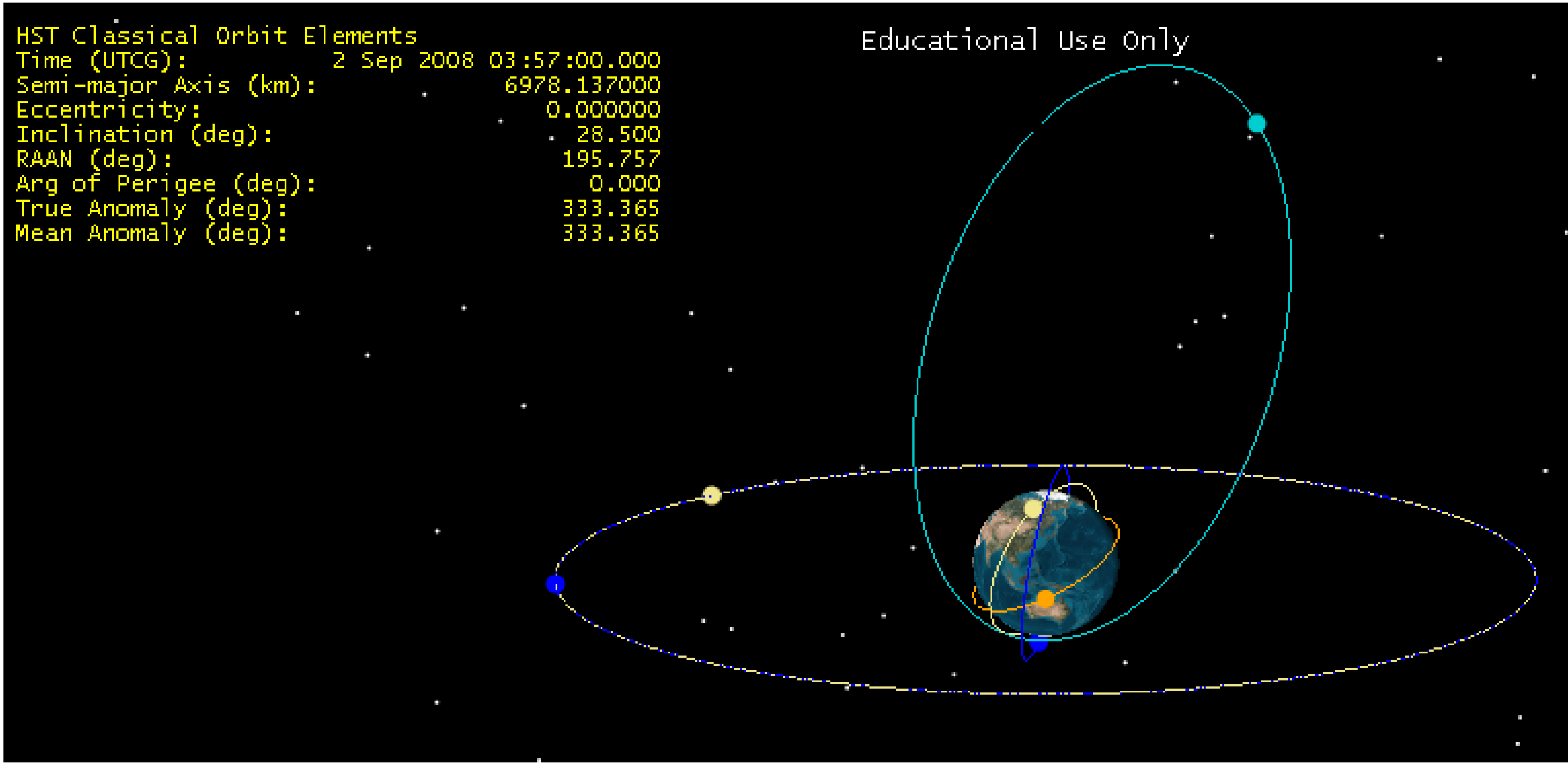
Satellite Orbits: Conclusions

Gentle introduction to satellite orbits; more details in the astrodynamics course.

Closed-form solution of the 2-body problem from which we deduced Kepler's laws.

Orbit perturbations cannot be ignored for accurate orbit propagation and for mission design.

Orbit transfers are commonly encountered. Satellite must often have their own propulsion.



METEOSAT 6-7, HST, OUFTI-1, SPOT-5, MOLNIYA

GEO	LEO	LEO	(LEO) SSO	Molniya
i=0	i=28.5	i=71	i=98.7	i=63.4