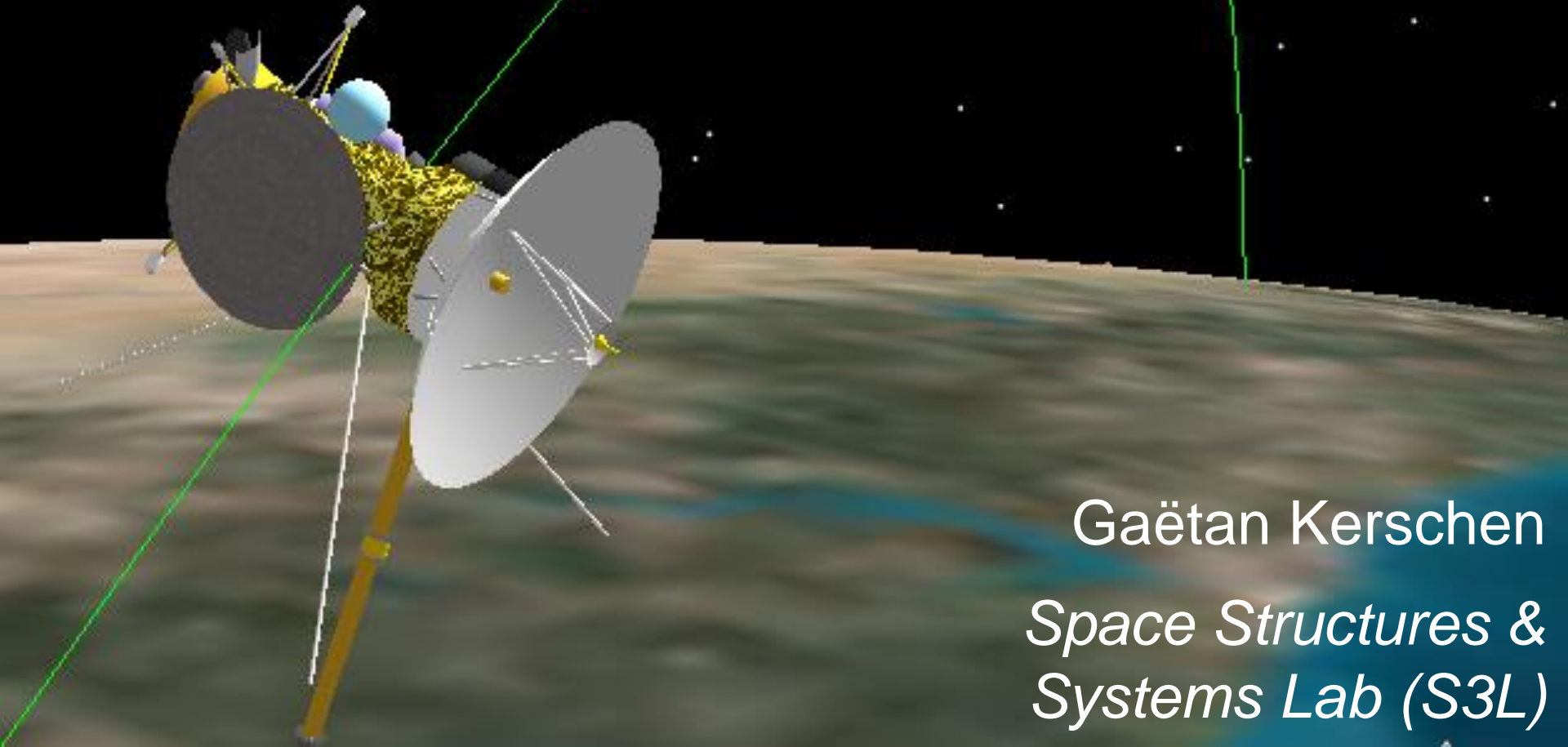


Cassini Classical Orbit Elements  
Time (UTCG): 15 Oct 1997 09:18:54.000  
Semi-major Axis (km): 6685.637000  
Eccentricity: 0.020566  
Inclination (deg): 30.000  
RAAN (deg): 150.546  
Arg of Perigee (deg): 230.000  
True Anomaly (deg): 136.530  
Mean Anomaly (deg): 134.891

# Aerodynamics (AERO0024)

## 5. Dominant Perturbations



Gaëtan Kerschen  
*Space Structures &  
Systems Lab (S3L)*

# Motivation



Assumption of a two-body system in which the central body acts gravitationally as a point mass.

In many practical situations, a satellite experiences significant perturbations (accelerations).

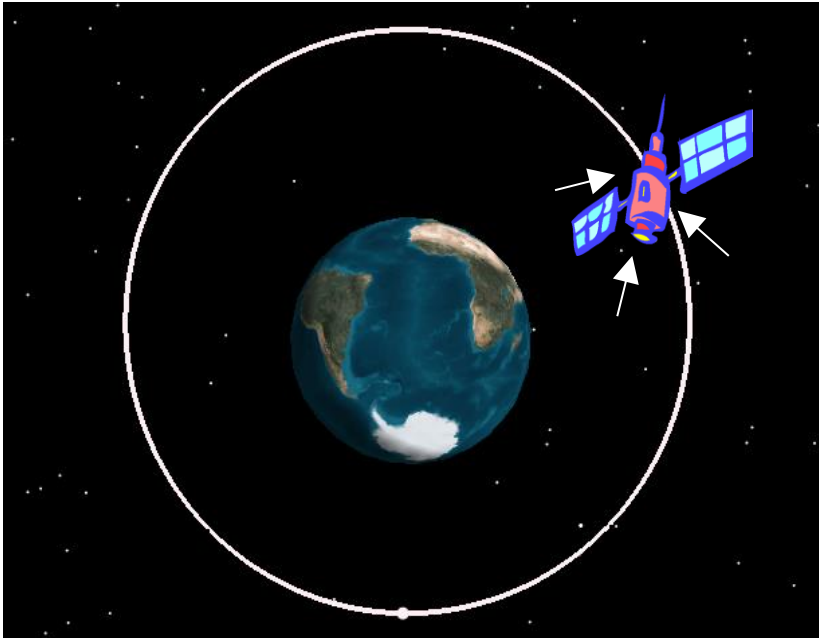
These perturbations are sufficient to cause predictions of the position of the satellite based on a Keplerian approach to be in significant error in a brief time.

# Different Perturbations and Importance ?

In low-earth orbit (LEO) ?

In geostationary orbit (GEO) ?

# Non-Keplerian Motion



## Dominant perturbations

Earth's gravity field

Atmospheric drag

Third-body perturbations

Solar radiation pressure

# Orders of Magnitude

400 kms

1000 kms

36000 kms

---

**Oblateness**

Drag

**Oblateness**

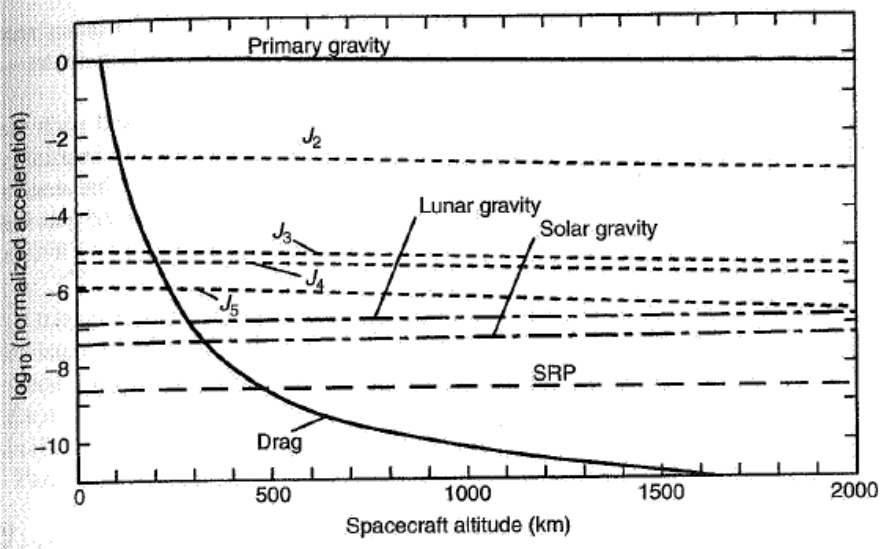
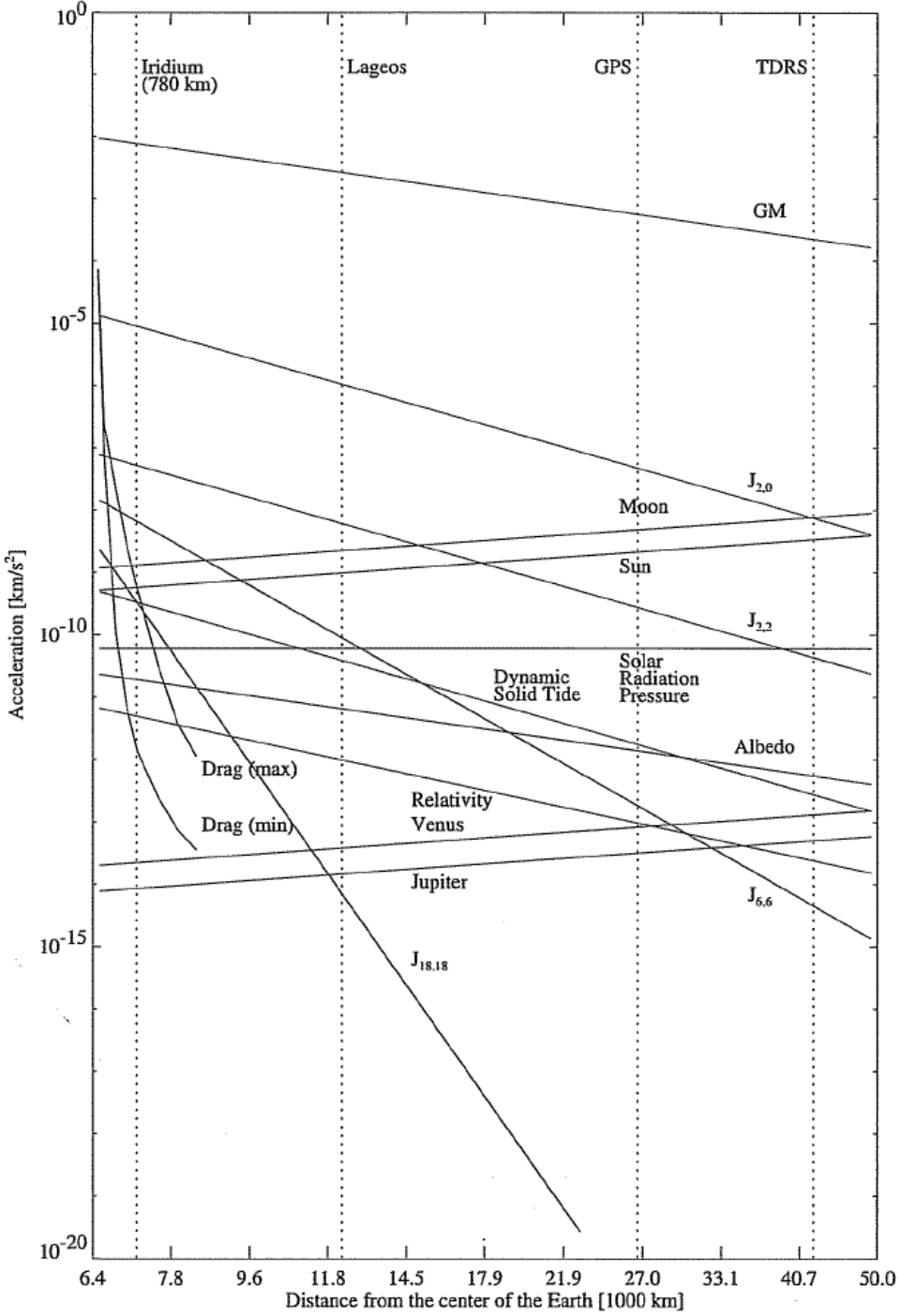
Sun and moon

Oblateness

Sun and moon

SRP

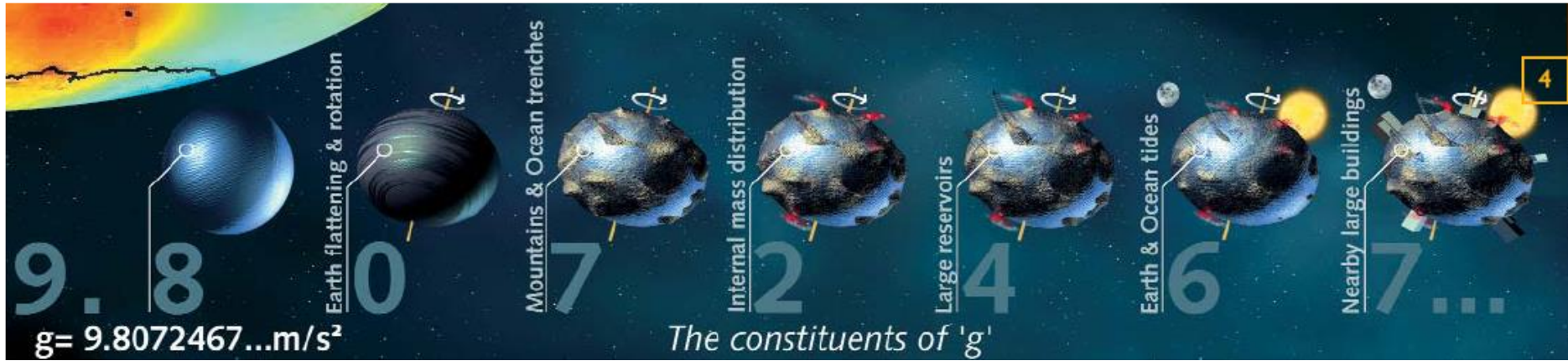
# Satellite dependent !



Montenbruck and Gill, *Satellite orbits*, Springer, 2000

Fortescue et al., *Spacecraft systems engineering*, 2003

# The Earth is not a Sphere...



# 1<sup>st</sup> Order Effect: Equatorial Bulge

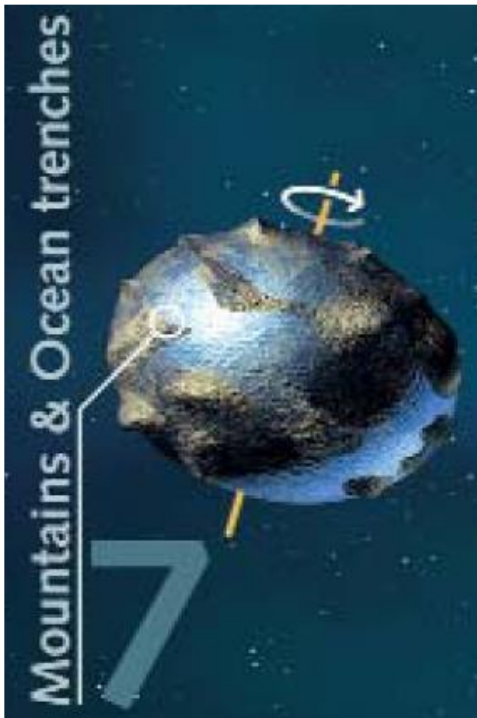


Because our planet rotates, the centrifugal force tends to pull material outwards around the Equator where the velocity of rotation is at its highest:

- ⇒ The Earth's radius is 21km greater at the Equator compared to the poles.
- ⇒ The force of gravity is weaker at the Equator ( $g=9.78 \text{ m/s}^2$ ) than it is at the poles ( $g=9.83 \text{ m/s}^2$ ).



# 2<sup>nd</sup> Order Effect: Mountains and Oceans



Rather than being smooth, the surface of the Earth is relatively “lumpy”:

- ⇒ There is about a 20 km difference in height between the highest mountain and the deepest part of the ocean floor.

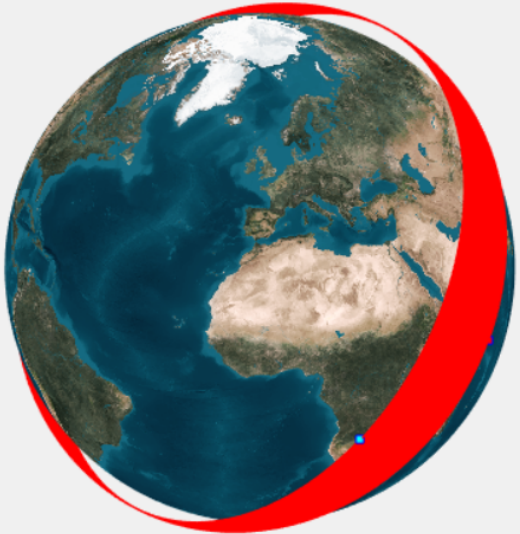
# 3<sup>rd</sup> Order Effect: Internal Mass Distribution



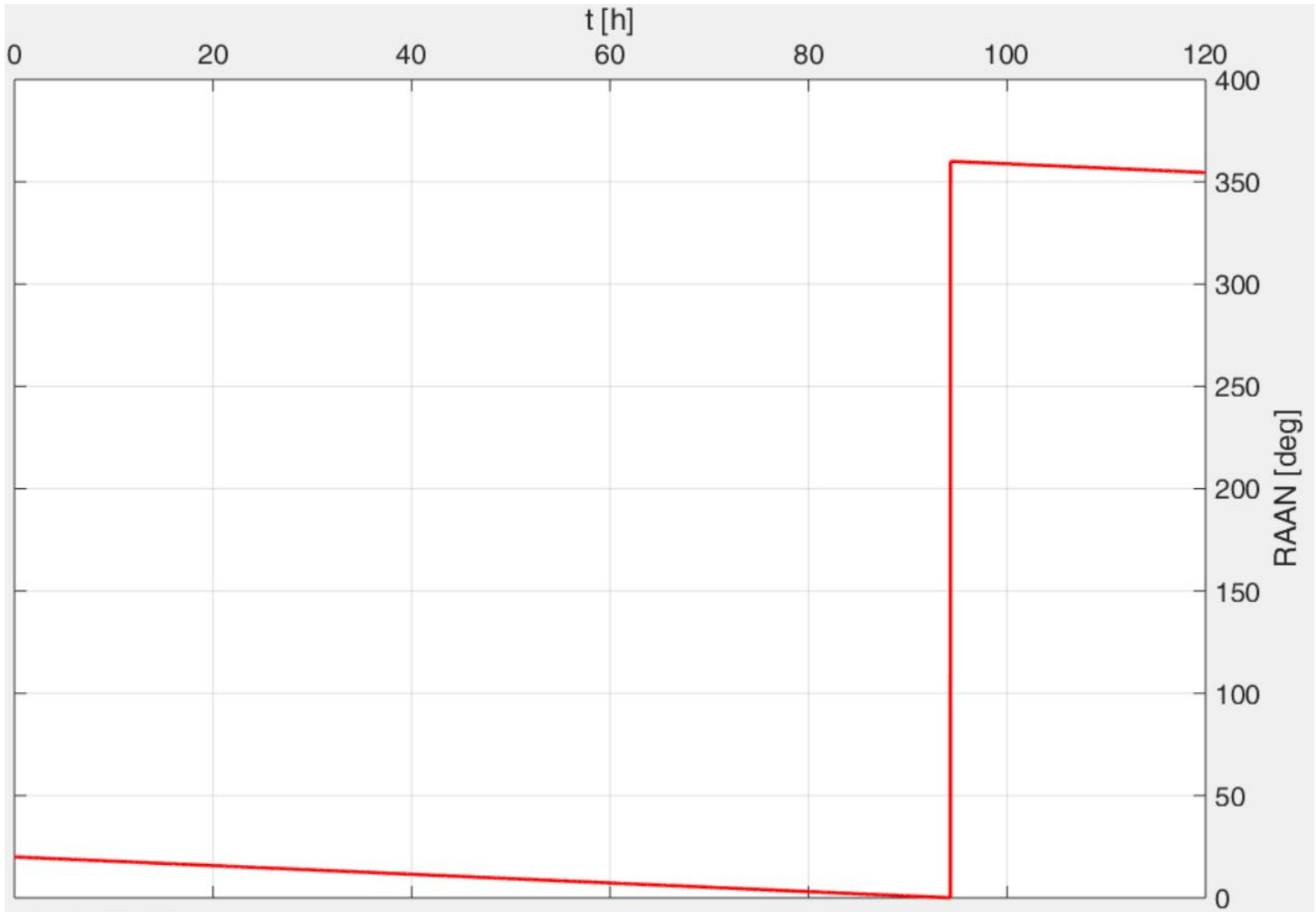
The different materials that make up the layers of the Earth's crust and mantle are far from homogeneously distributed:

- ⇒ For instance, the crust beneath the oceans is a lot thinner and denser than the continental crust.

# The Effect of Earth Oblateness

<b>Keplerian Parameters</b>	
Semi-major axis [m]	6778e3
Eccentricity	0.0
Inclination [deg]	51
Argument of perigee [deg]	0.0
RAAN [deg]	20
True anomaly [deg]	0.0
<b>Control</b>	
<input type="radio"/> None <input type="radio"/> Cross-Section <input type="radio"/> Attitude	
<b>Force Model</b>	
<input checked="" type="checkbox"/> Non-spherical <input type="checkbox"/> Drag <input type="checkbox"/> SRP <input type="checkbox"/> Third-body Sun <input type="checkbox"/> Third-body Sun	
<b>ECI to ECEF</b>	
<input type="checkbox"/> Precession <input type="checkbox"/> Nutation <input type="checkbox"/> Polar Wandering <input type="button" value="Simplified"/>	
<b>Spacecraft Properties</b>	
Mass [kg]	4
Sizes [m, m, m]	[0.3, 0.1, 0.1]
Cross-section to TAS [m <sup>2</sup> ]	0.03
Cross-section to Sun [m <sup>2</sup> ]	0.03
Drag Coefficient	4
Reflectivity Coefficient	[1.2, 1.2, 1.2, 1.2, 1.2, 1.2]
<b>Density Model</b>	
<input type="radio"/> Harris-Priester <input type="radio"/> Jacchia 71 <input checked="" type="radio"/> Jacchia-Roberts <input type="checkbox"/> Measured data	
<b>Density Parameters</b>	
Harris-Priester coeff.	0
DailyF10.7	155
Averaged F10.7	155
Geomagnetic activity	3
<b>Gravity Model</b>	
Maximum Degree	2
Maximum Order	0
<input type="checkbox"/> Download Data	
<b>Date</b>	
Year	2010
Month	10
Day	23
Hours	19
Minutes	40
Seconds	00
Simulation time [s]	5*24 * 3600
<b>Integration Parameters</b>	
Relative tolerance	1e-13
Absolute tolerance	1e-13
Output time step [s]	60
<b>Orbit 3D</b>	
	
<input type="button" value="RUN!"/>	

# The Effect of Earth Oblateness

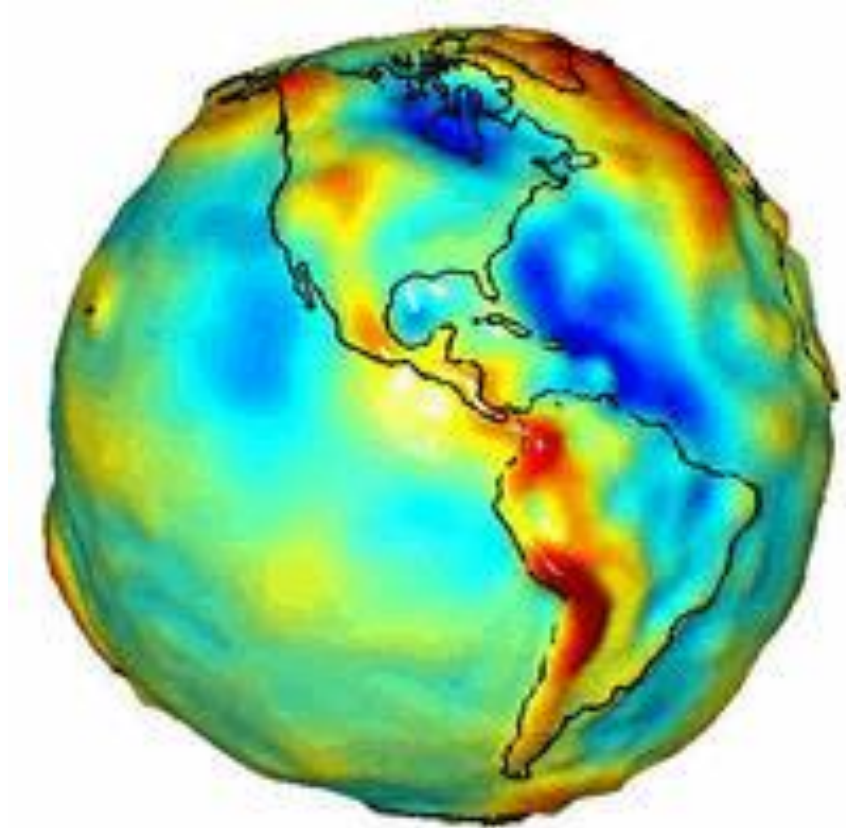


# The True Figure of the Earth

The **geoid** is that equipotential surface which would coincide exactly with the mean ocean surface of the Earth, if the oceans were in equilibrium, at rest, and extended through the continents:

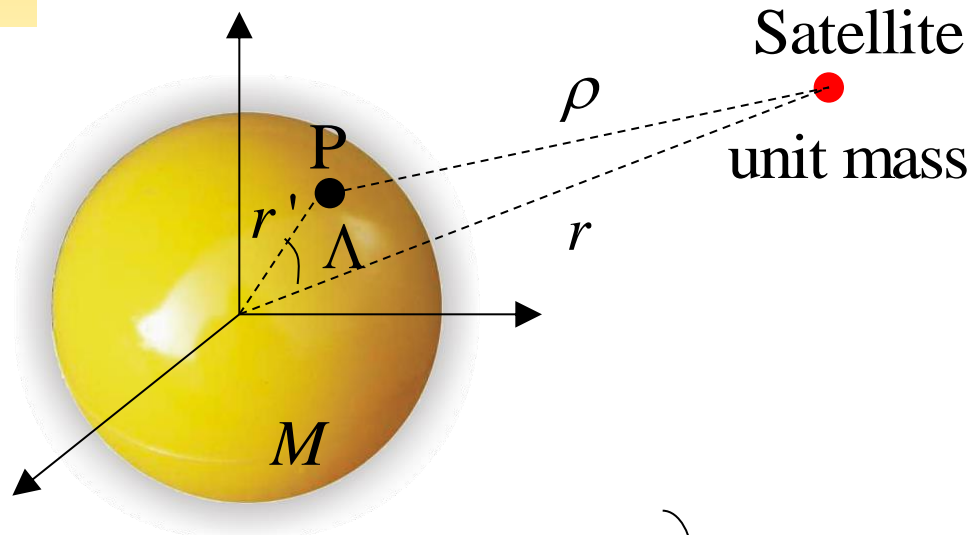
- ⇒ It is by definition a surface to which the force of gravity is everywhere perpendicular.
- ⇒ It is an irregular surface but considerably smoother than Earth's physical surface. While the latter has excursions of almost 20 km, the total variation in the geoid is less than 200 m.

# The True Figure of the Earth



How to model the gravitational potential accurately ?

# Mathematical Modeling



$$r = \sqrt{x^2 + y^2 + z^2}$$

$$r' = \sqrt{\xi^2 + \eta^2 + \zeta^2}$$

$U = -V$ , potential,  $\ddot{\mathbf{r}} = \nabla U$

$V$ , potential energy

$$U = G \int_{body} \frac{dm}{\rho}$$

$$\rho = \sqrt{r^2 + r'^2 - 2r'r \cos \Lambda}$$

$$\cos \Lambda = \frac{\mathbf{r} \cdot \mathbf{r}'}{r \cdot r'} \quad \alpha = \frac{r'}{r} < 1$$

$$U = G \int_{body} \frac{dm}{r \sqrt{1 - 2\alpha \cos \Lambda + \alpha^2}}$$

# Legendre Polynomials

First introduced in 1782 by Legendre

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$$

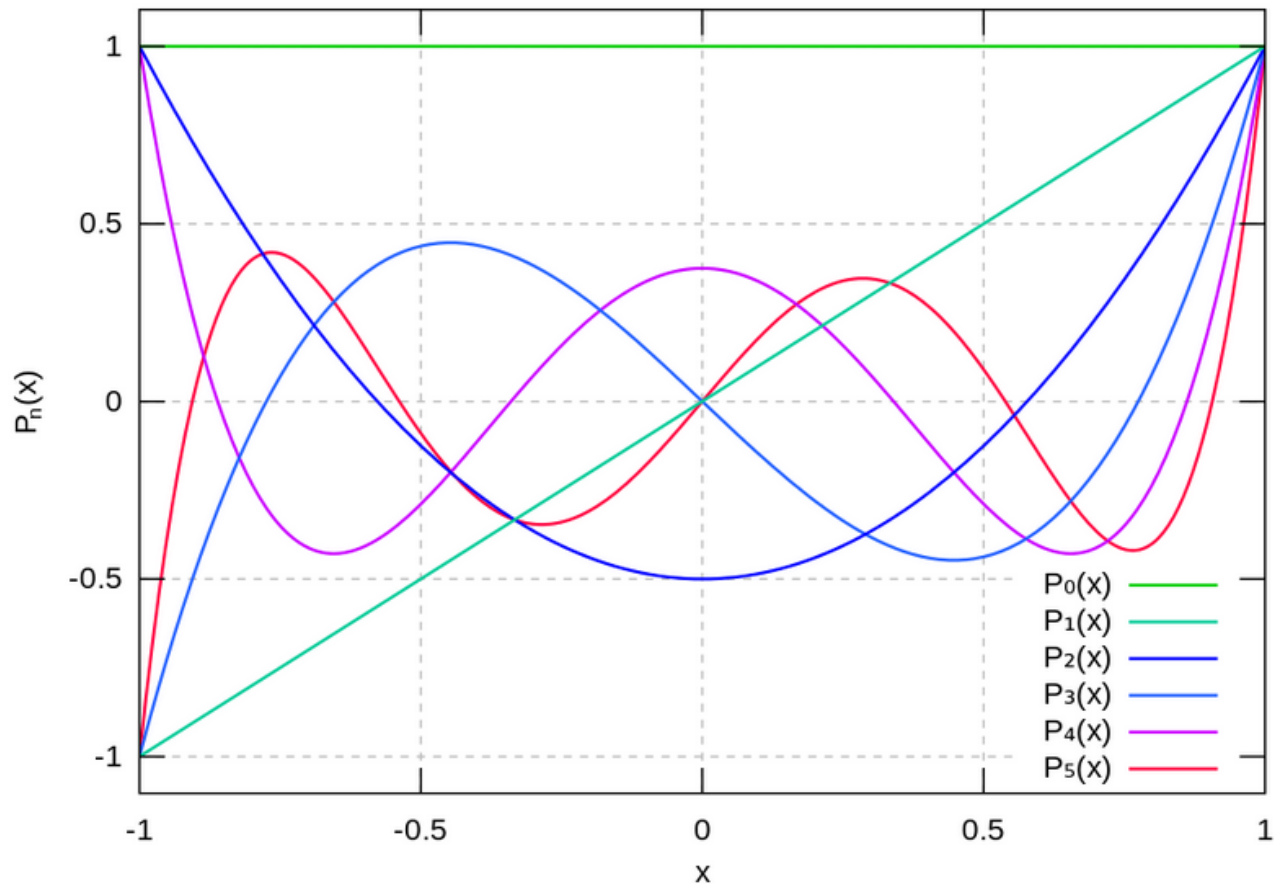
$n$	$P_n(x)$
0	1
1	$x$
2	$\frac{1}{2} (3x^2 - 1)$
3	$\frac{1}{2} (5x^3 - 3x)$
4	$\frac{1}{8} (35x^4 - 30x^2 + 3)$

$$\frac{1}{\sqrt{1 - 2xt + t^2}} = \sum_{n=0}^{\infty} P_n(x)t^n$$



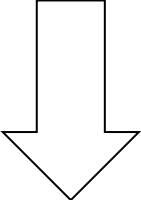
# Legendre Polynomials Are Orthogonal

$$\int_{-1}^1 P_m(x)P_n(x) dx = 0 \quad \text{if } n \neq m.$$



# Let's Use Them

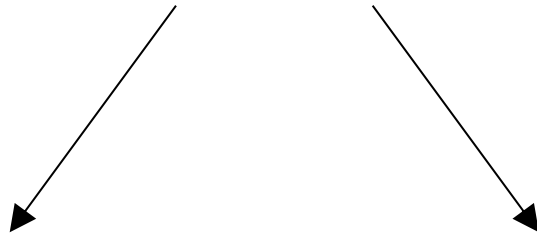
$$U = G \int_{body} \frac{dm}{r \sqrt{1 - 2\alpha \cos \Lambda + \alpha^2}^l}$$


$$\frac{1}{\sqrt{1 - 2xt + t^2}} = \sum_{n=0}^{\infty} P_n(x) t^n$$

$$U = \frac{G}{r} \int_{body} \sum_{l=0}^{\infty} \alpha^l P_l[\cos(\Lambda)] dm$$

# Summing Up...

$$U = \frac{G}{r} \int_{body} \sum_{l=0}^{\infty} \alpha^l P_l[\cos(\Lambda)] dm$$



Geometric method (intuitive feel for gravity and inertia)

$$U = U_0 + U_1 + U_2 + \dots$$

*Theory*

Spherical-harmonic expansion

*Experiments*

# Geometric Method: First and Second Terms

$$U_0 = \frac{G}{r} \int dm = \frac{\mu}{r} \quad \text{Two-body potential}$$

$$\begin{aligned} U_1 &= \frac{G}{r} \int \cos(\Lambda) \alpha \, dm = \frac{G}{r} \int \frac{x\xi + y\eta + z\zeta}{r^2} \, dm \\ &= \frac{G}{r^3} \left( x \int \xi \, dm + y \int \eta \, dm + z \int \zeta \, dm \right) = 0 \end{aligned}$$

Center of mass at the origin of  
the coordinate frame

## Geometric Method: Third Term

$$\begin{aligned}U_2 &= \frac{G}{r} \int \frac{\alpha^2}{2} (3 \cos^2 \Lambda - 1) dm \\&= \frac{G}{2r^3} \int 2r'^2 dm - \frac{G}{2r^3} \int 3r'^2 \sin^2 \Lambda dm \\&= \frac{G}{2r^3} (A + B + C - 3I)\end{aligned}$$

$$\left( \begin{aligned} \int 2r'^2 dm &= \int (\eta^2 + \zeta^2) dm + \int (\xi^2 + \zeta^2) dm + \int (\eta^2 + \xi^2) dm \\ &= A + B + C \quad \text{Moments of inertia} \end{aligned} \right)$$

$$\left( \int r'^2 \sin^2 \Lambda dm = I \quad \text{Polar moment of inertia} \right)$$

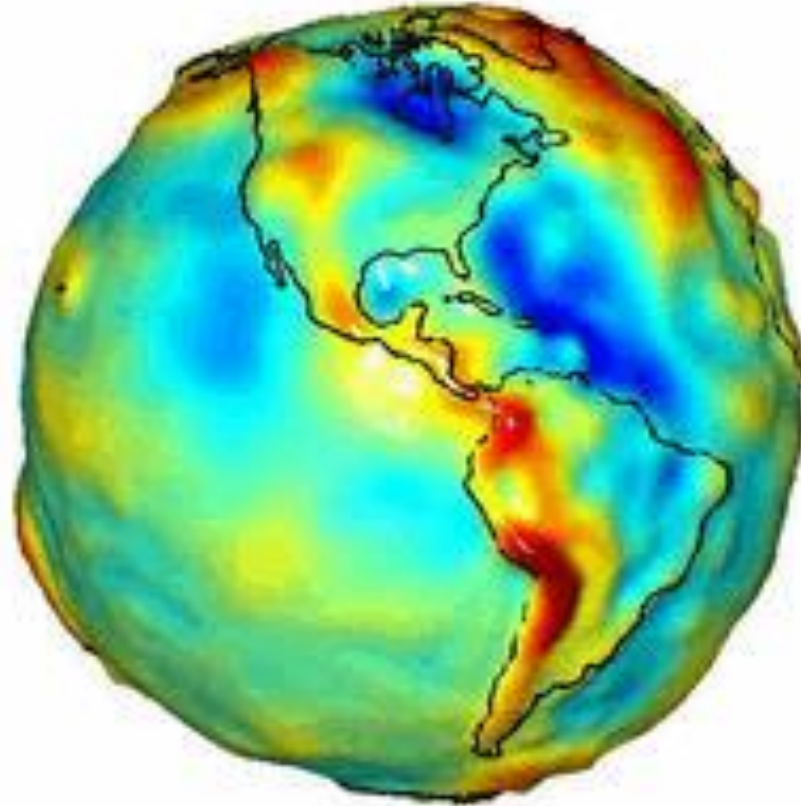
# Geometric Method: MacCullagh's Formula

$$U = \frac{Gm_{\oplus}}{r} + \frac{G}{2r^3} (A + B + C - 3I) + \dots$$

Some of the simplest assumptions are

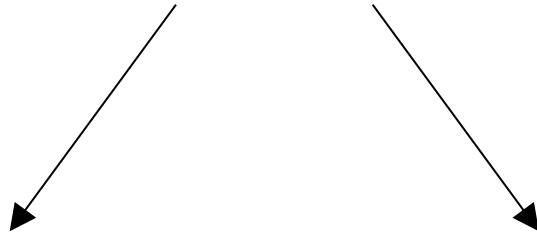
- the ellipsoidal Earth (oblate spheroid) with uniform density ( $a=b>c$ ).
- triaxial ellipsoid ( $a>b>c$ ).

# Geometric Method: Difficult to Go Further...



# Summing up...

$$U = \frac{G}{r} \int_{body} \sum_{l=0}^{\infty} \alpha^l P_l[\cos(\Lambda)] dm$$



Geometric method (intuitive feel for gravity and inertia)

$$U = U_0 + U_1 + U_2 + \dots$$

*Theory*

Spherical-harmonic expansion

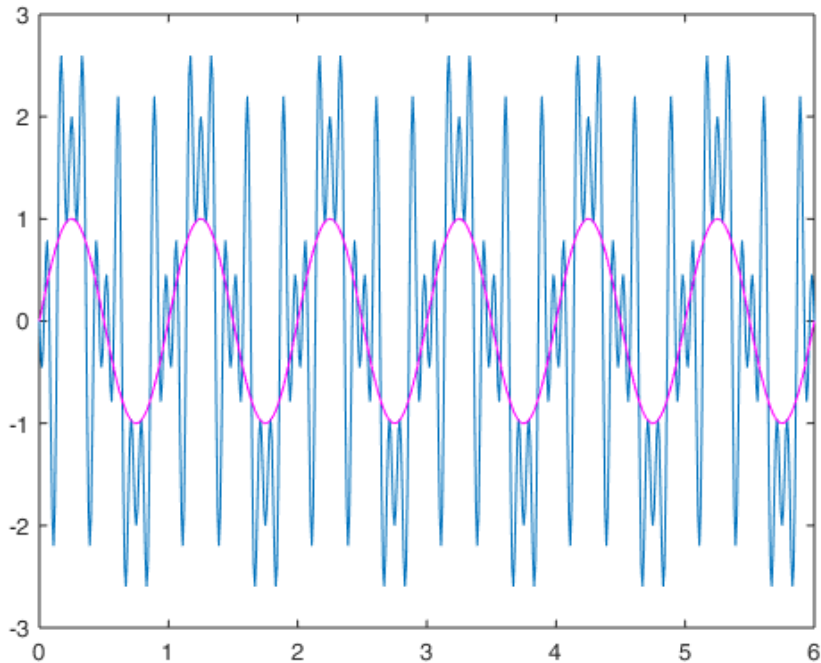
*Experiments*



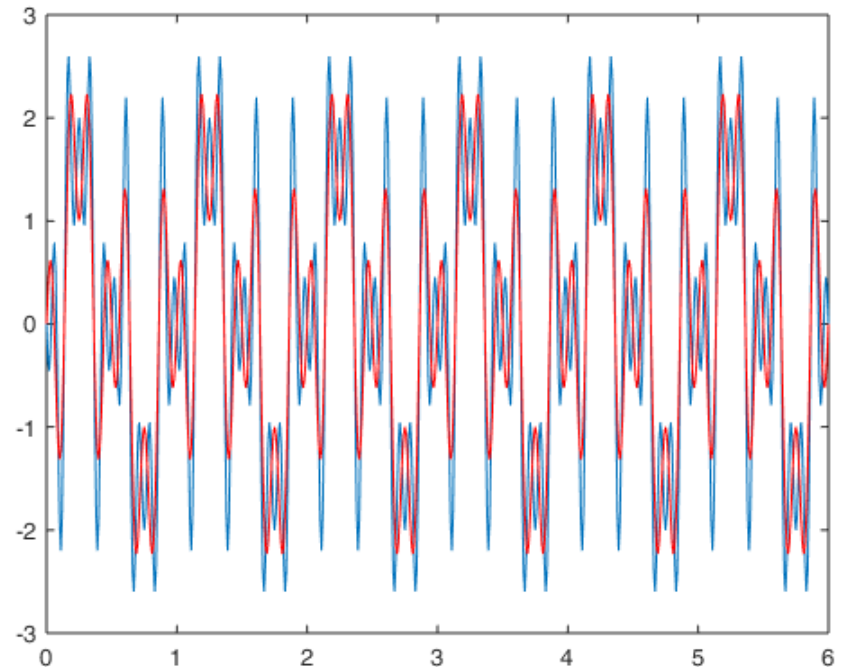
# Philosophy of the modeling

Modeling a complex time series with sine functions

Modeling with 1 sine



Modeling with 3 sines



# Matlab example

```
clear all;
close all

temps=[0:0.01:6];

TimeSeries=sin(2*pi*temps)-sin(3*2*pi*temps)+sin(7*2*pi*temps)-sin(11*2*pi*temps);
plot(temps,TimeSeries);pause

A=sin(2*pi*temps)';
b=TimeSeries';
x=pinv(A)*b;

Fitting1Sine=A*x;hold on;plot(temps,Fitting1Sine,'m');pause

A=[sin(2*pi*temps)' sin(3*2*pi*temps)'];
b=TimeSeries';
x=pinv(A)*b;

Fitting1Sine=A*x;hold on;plot(temps,Fitting1Sine,'k');pause

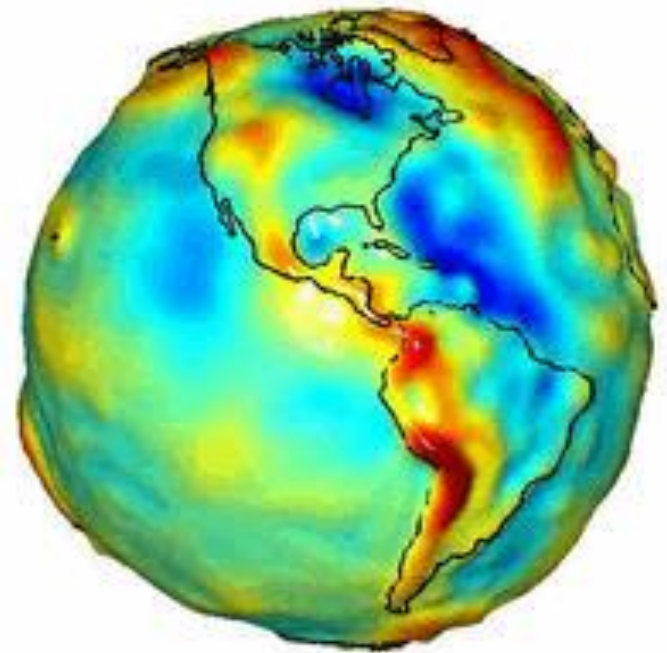
A=[sin(2*pi*temps)' sin(3*2*pi*temps)' sin(7*2*pi*temps)'];
b=TimeSeries';
x=pinv(A)*b;

Fitting1Sine=A*x;hold on;plot(temps,Fitting1Sine,'r');
```

# Spherical harmonics

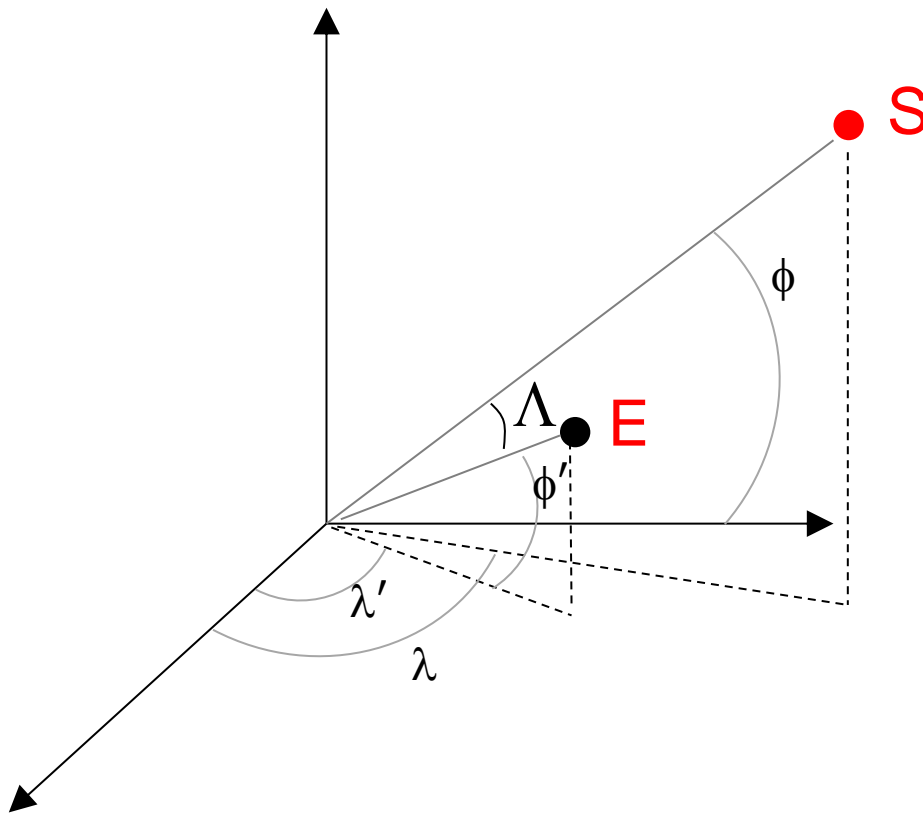
A set of functions used to represent functions on the surface of the sphere. They are a higher-dimensional analogy of Fourier series.

So any object that looks « kind-of-spherical » can be decomposed into an infinite sum of basic functions, as long as you multiply each basic function by the right coefficient



Our objective !

# Spherical Trigonometry



$\phi \rightarrow$  latitude sat  
 $\lambda \rightarrow$  longitude sat  
 $\phi' \rightarrow$  latitude Earth  
 $\lambda' \rightarrow$  longitude Earth

$$\cos \Lambda = \cos(90 - \phi') \cos(90 - \phi) + \sin(90 - \phi') \sin(90 - \phi) \cos(\lambda - \lambda')$$

# Addition Theorem for Spherical Harmonics

If  $\cos \Lambda = \cos(90 - \phi') \cos(90 - \phi) + \sin(90 - \phi') \sin(90 - \phi) \cos(\lambda - \lambda')$

Then,

$$P_l(\cos \Lambda) = \sum_{m=0}^l (2 - \delta_{0m}) \frac{(l-m)!}{(l+m)!} P_{lm}(\sin \phi) P_{lm}(\sin \phi') \cos(m(\lambda - \lambda'))$$

where  $P_{lm}(u) = (1 - u^2)^{m/2} \frac{d}{du^m} P_n(u)$

Associated Legendre polynomial of degree  $l$  and order  $m$

$$U = \frac{G}{r} \int_{body} \sum_{l=0}^{\infty} \alpha^l P_l(\cos \Lambda) dm = \frac{G}{r} \int_{body} \sum_{l=0}^{\infty} \left(\frac{r'}{r}\right)^l P_l(\cos \Lambda) dm$$

$$P_l(\cos \Lambda) = \sum_{m=0}^l (2 - \delta_{0m}) \frac{(l-m)!}{(l+m)!} P_{lm}(\sin \phi) P_{lm}(\sin \phi') \cos(m(\lambda - \lambda'))$$

$\swarrow$   
 $(\cos m\lambda \cos m\lambda' + \sin m\lambda \sin m\lambda')$



*Depends only on the satellite  $(r, \phi, \lambda)$*

$$U = \frac{GM_{\oplus}}{r} \left\{ \sum_{l=0}^{\infty} \sum_{m=0}^l \left(\frac{R_{\oplus}}{r}\right)^l P_{lm}(\sin \phi) (C_{lm} \cos m\lambda + S_{lm} \sin m\lambda) \right\}$$

*Depends only on the Earth  $(\phi', \lambda')$ : spherical harmonics*

$$C_{lm} = \frac{(2 - \delta_{0m}) (l-m)!}{M_{\oplus} (l+m)!} \int \left(\frac{r'}{R_{\oplus}}\right)^l P_{lm}(\sin \phi') \cos m\lambda' dm$$

$$S_{lm} = \frac{(2 - \delta_{0m}) (l-m)!}{M_{\oplus} (l+m)!} \int \left(\frac{r'}{R_{\oplus}}\right)^l P_{lm}(\sin \phi') \sin m\lambda' dm$$

# Normalization: End Result

$$U = \frac{GM_{\oplus}}{r} \left\{ \sum_{l=0}^{\infty} \sum_{m=0}^l \left( \frac{R_{\oplus}}{r} \right)^l \bar{P}_{lm}(\sin \phi) (\bar{C}_{lm} \cos m\lambda + \bar{S}_{lm} \sin m\lambda) \right\}$$

$$\begin{Bmatrix} \bar{C}_{lm} \\ \bar{S}_{lm} \end{Bmatrix} = \sqrt{\frac{(l+m)!}{(2-\delta_{0m})(2n+1)(l-m)!}} \begin{Bmatrix} C_{lm} \\ S_{lm} \end{Bmatrix}$$

$$\bar{P}_{lm} = \sqrt{\frac{(2-\delta_{0m})(2n+1)(l-m)!}{(l+m)!}} P_{lm}$$

# Very Important Remark

Many different expressions exist in the literature:

$$\Rightarrow V = \pm V$$

$$\Rightarrow P_l^m = (-1)^m P_{lm}$$

$\Rightarrow$  Normalized or non-normalized coefficients

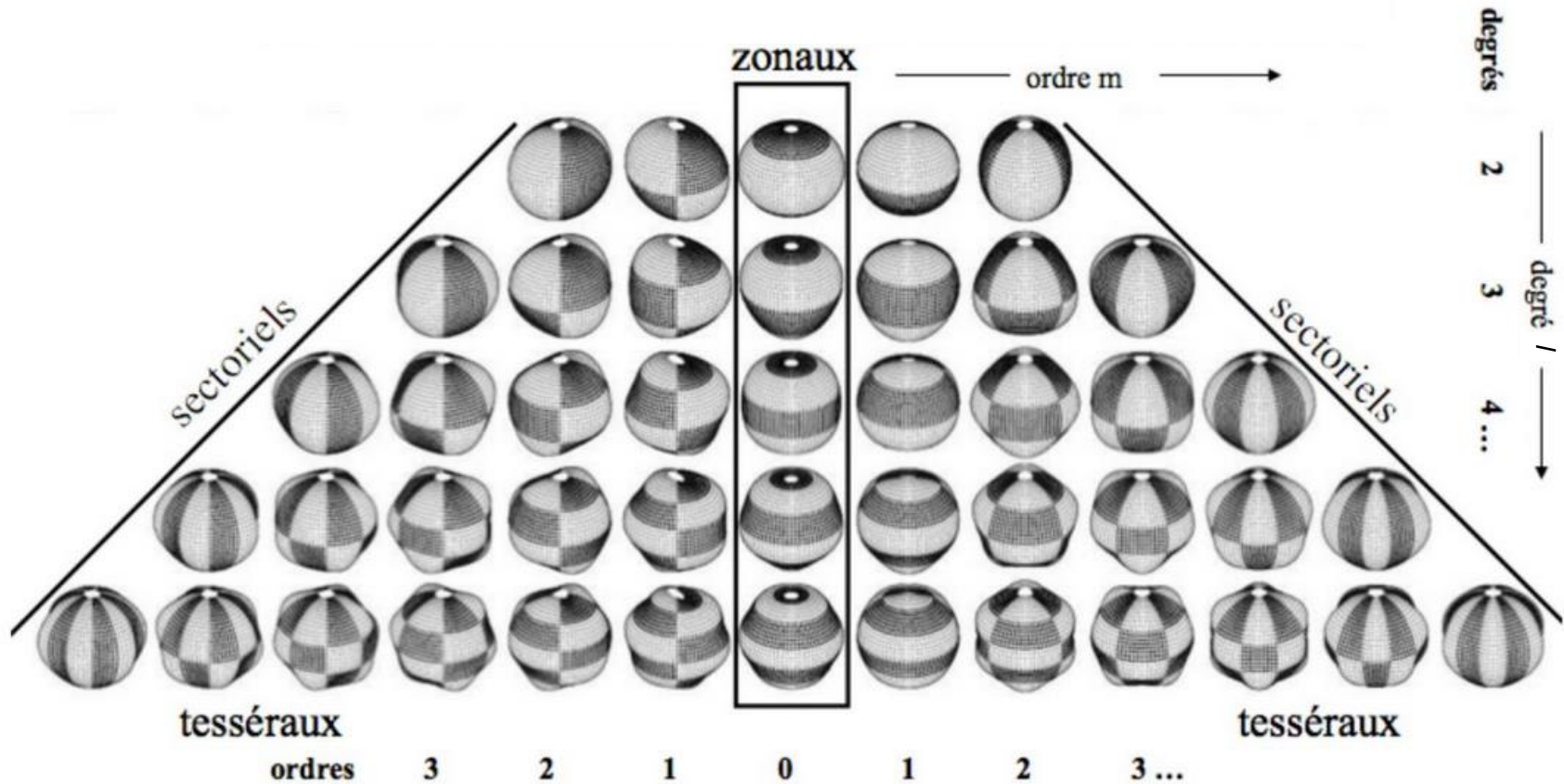
$\Rightarrow$  Latitude or colatitude ( $\sin \phi$  or  $\cos \phi$ )

$\Rightarrow \dots$

Be always aware of the conventions/definitions used !



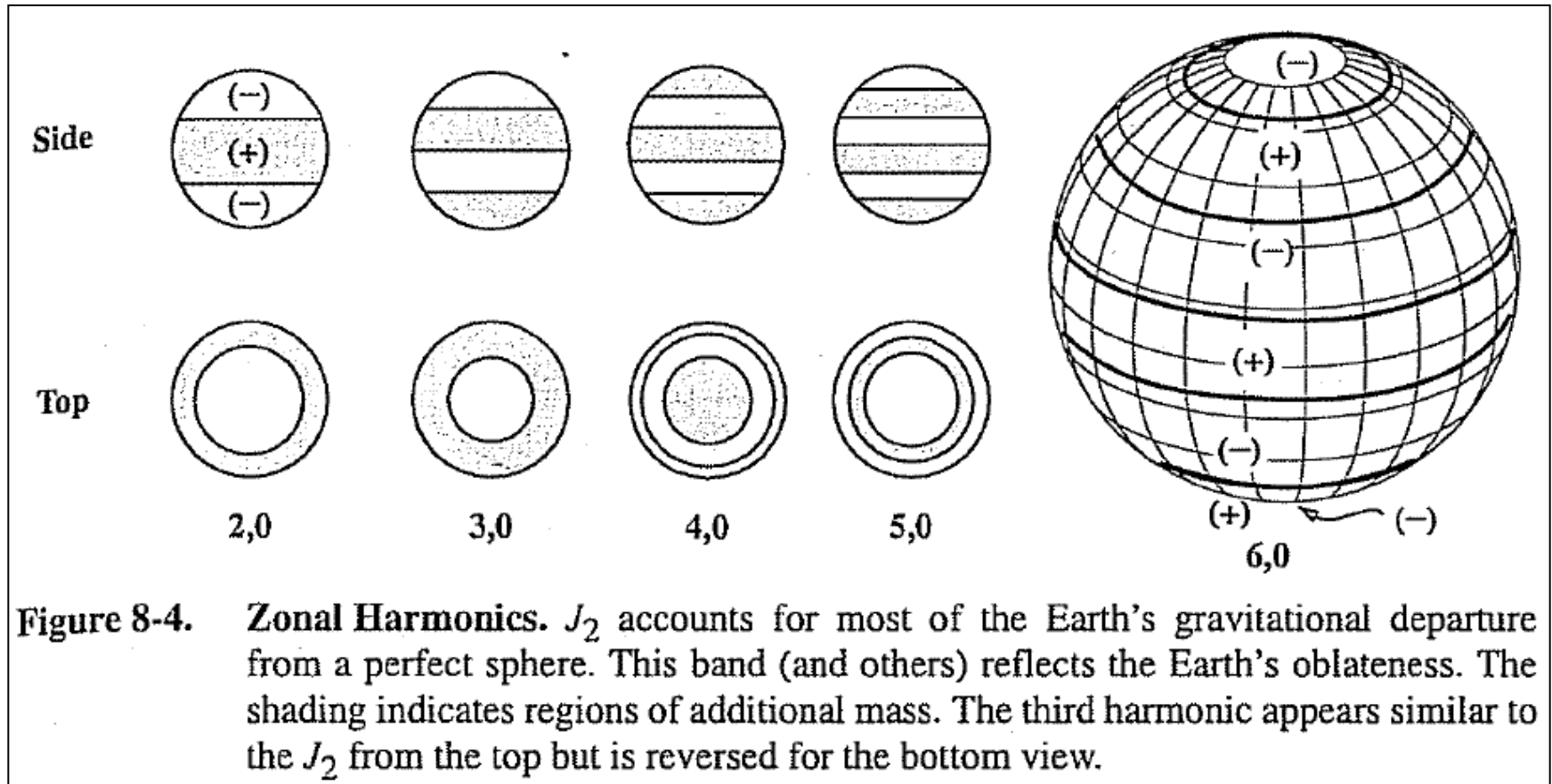
# Spherical Harmonics



The degree «  $l$  » is the total number of waves. The order «  $m$  » is the number of waves in longitude. The number of waves in latitude is thus «  $l - m$  ».

# Zonal Harmonics ( $m=0$ )

Each boundary is a root of the Legendre polynomial.



Vallado, *Fundamental of Astrodynamics and Applications*, Kluwer, 2001.

# Zonal Harmonics (m=0)

The zonal coefficients are independent of longitude (symmetry with respect to the rotation axis).

$$U = \frac{\mu}{r} \left\{ 1 + \sum_{l=2}^{\infty} \sum_{m=0}^l \left( \frac{R_{\oplus}}{r} \right)^l \bar{P}_{lm} [\sin \phi_{sat}] \left[ \bar{C}_{l,m} \cos(m\lambda_{sat}) + \bar{S}_{l,m} \sin(m\lambda_{sat}) \right] \right\}$$

$$\begin{array}{l} \Downarrow \\ J_l = -C_{l,0} \\ S_{l,0} = 0 \text{ (definition)} \end{array}$$

$$U = \frac{\mu}{r} \left\{ 1 + \sum_{l=2}^{\infty} \left( \frac{R_{\oplus}}{r} \right)^l \left[ -J_l P_l [\sin \phi_{sat}] + \sum_{m=1}^l \bar{P}_{lm} [\sin \phi_{sat}] \left[ \bar{C}_{l,m} \cos(m\lambda_{sat}) + \bar{S}_{l,m} \sin(m\lambda_{sat}) \right] \right] \right\}$$

# EGM96

0,1 ?

Degree and Order		Normalized Gravitational Coefficients	
n	m	$\bar{C}_{nm}$	$\bar{S}_{nm}$
2	0	-.484165371736E -03	
2	1	-.186987635955E -09	.119528012031E-08
2	2	.243914352398E -05	-.140016683654E-05
3	0	.957254173792E -06	
3	1	.202998882184E -05	.248513158716E-06
3	2	.904627768605E -06	-.619025944205E-06
3	3	.721072657057E -06	.141435626958E-05
4	0	.539873863789E -06	
4	1	-.536321616971E -06	-.473440265853E-06
4	2	.350694105785E -06	.662671572540E-06
4	3	.990771803829E -06	-.200928369177E-06
4	4	-.188560802735E -06	.308853169333E-06
5	0	.685323475630E -07	
5	1	-.621012128528E -07	-.944226127525E-07
5	2	.652438297612E -06	-.323349612668E-06
5	3	-.451955406071E -06	-.214847190624E-06
5	4	-.295301647654E -06	.496658876769E-07
5	5	.174971983203E -06	-.669384278219E-06

# First Zonal Harmonic: J2,0 or J2

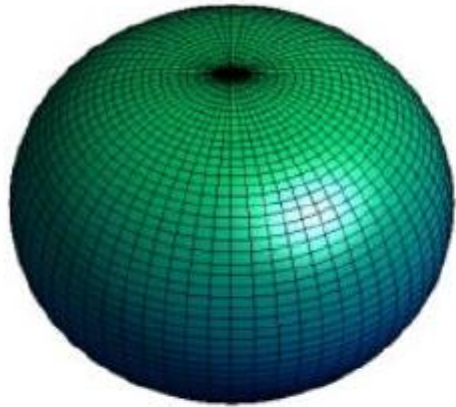
It represents the Earth's equatorial bulge and quantifies the major effects of oblateness on orbits.

It is almost a thousand times as large as any of the other coefficients.

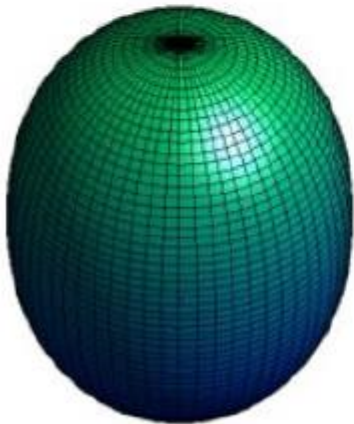
$$J_2 = -C_{2,0} = \sqrt{\frac{2.1.(2.2+1)}{2}} 0.4841 \times 10^{-3} = 0.001082$$

Degree and Order		Normalized Gravitat
n	m	$\bar{C}_{nm}$
2	0	-.484165371736E -03
2	1	-.186987635955E -09
2	2	2.12014252208E -05

# First Zonal Harmonic: $J_{2,0}$ or $J_2$



Oblate planet:  $J_2 > 0$



Prolate planet:  $J_2 < 0$

# Calculation of the Rotational Flattening

Equilibrium of a rotating self gravitating fluidlike body  
(uniform density)

<http://farside.ph.utexas.edu/teaching/336k/Newton/node109.html>

$$\frac{R_e - R_p}{R} = \frac{5\Omega^2 R^3}{4GM} \quad R \text{ is the mean radius}$$

$$\frac{R_e - R_p}{R} = \frac{5(7.27 \times 10^{-5})^2 (6.37 \times 10^6)^3}{4 \cdot 6.67 \times 10^{-11} \cdot 5.97 \times 10^{24}} = 0.0043$$

$$\frac{R_e - R_p}{R} = 0.0043 \times 6.37 \times 10^6 = 27\text{km vs } 21\text{km}$$

# First Zonal Harmonic of Other Planets

Planet	$J_2$
Mercury	60e-6
Venus	4.46e-6
Earth	1.08e-3
Moon	2.03e-4
Jupiter	1.47e-2
Saturn	1.63e-2

$$\frac{R_e - R_p}{R} = \frac{5\Omega^2 R^3}{4GM}$$

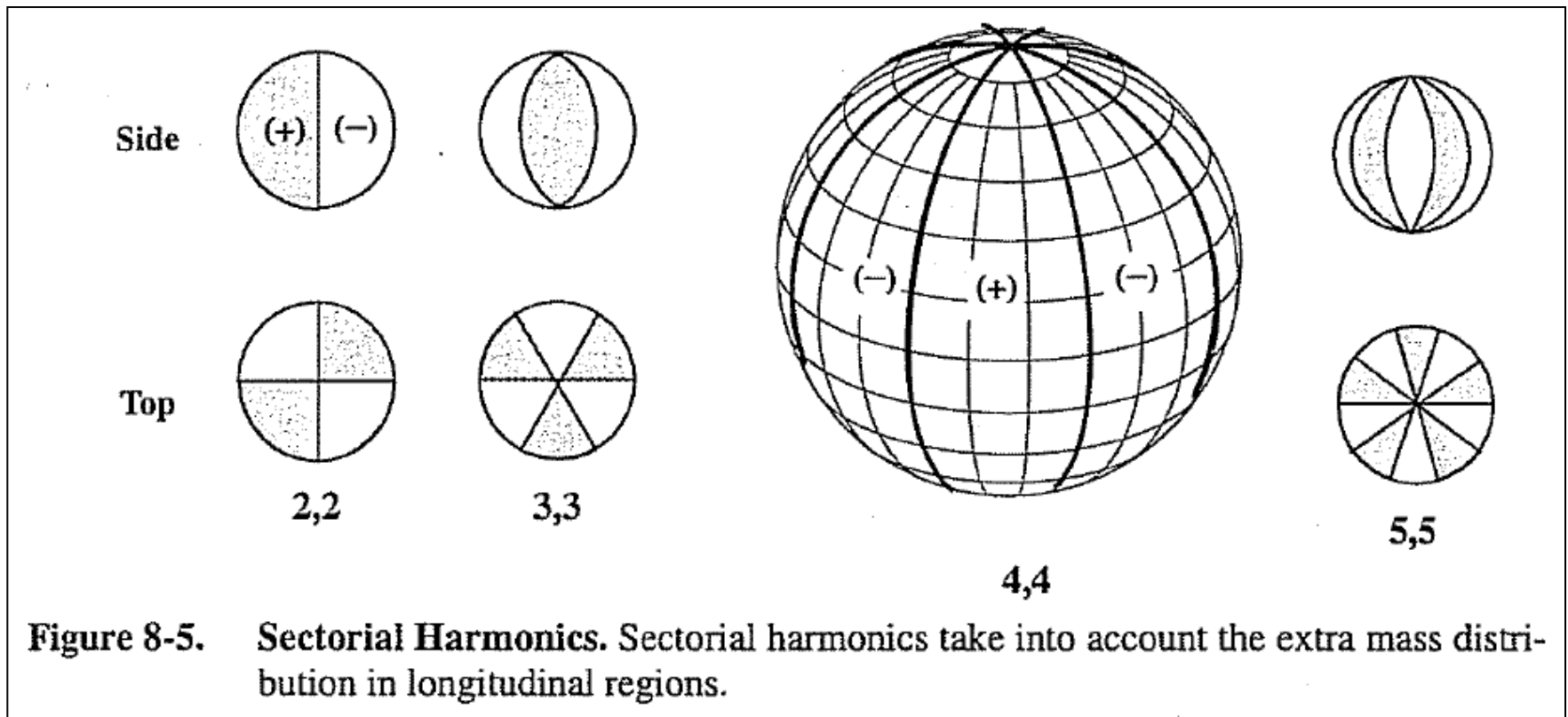
Celestial objects	Rotation period
Sun	25.379995 days (Carrington rotation) 35 days (high latitude)
Mercury	58.6462 days <sup>[7]</sup>
Venus	-243.0187 days <sup>[7][8]</sup>
Earth	0.99726968 days <sup>[7][9]</sup>
Moon	27.321661 days <sup>[10]</sup> (synchronous toward Earth)
Mars	1.02595675 days <sup>[7]</sup>
Ceres	0.37809 days <sup>[11]</sup>
Jupiter	0.4135344 days (deep interior) <sup>[12]</sup> 0.41007 days (equatorial) 0.41369942 days (high latitude)
Saturn	0.44403 days (deep interior) <sup>[12]</sup> 0.426 days (equatorial) 0.443 days (high latitude)
Uranus	-0.71833 days <sup>[7][8][12]</sup>
Neptune	0.67125 days <sup>[7][12]</sup>
Pluto	-6.38718 days <sup>[7][8]</sup> (synchronous with Charon)



# Sectorial Harmonics ( $l=m$ )

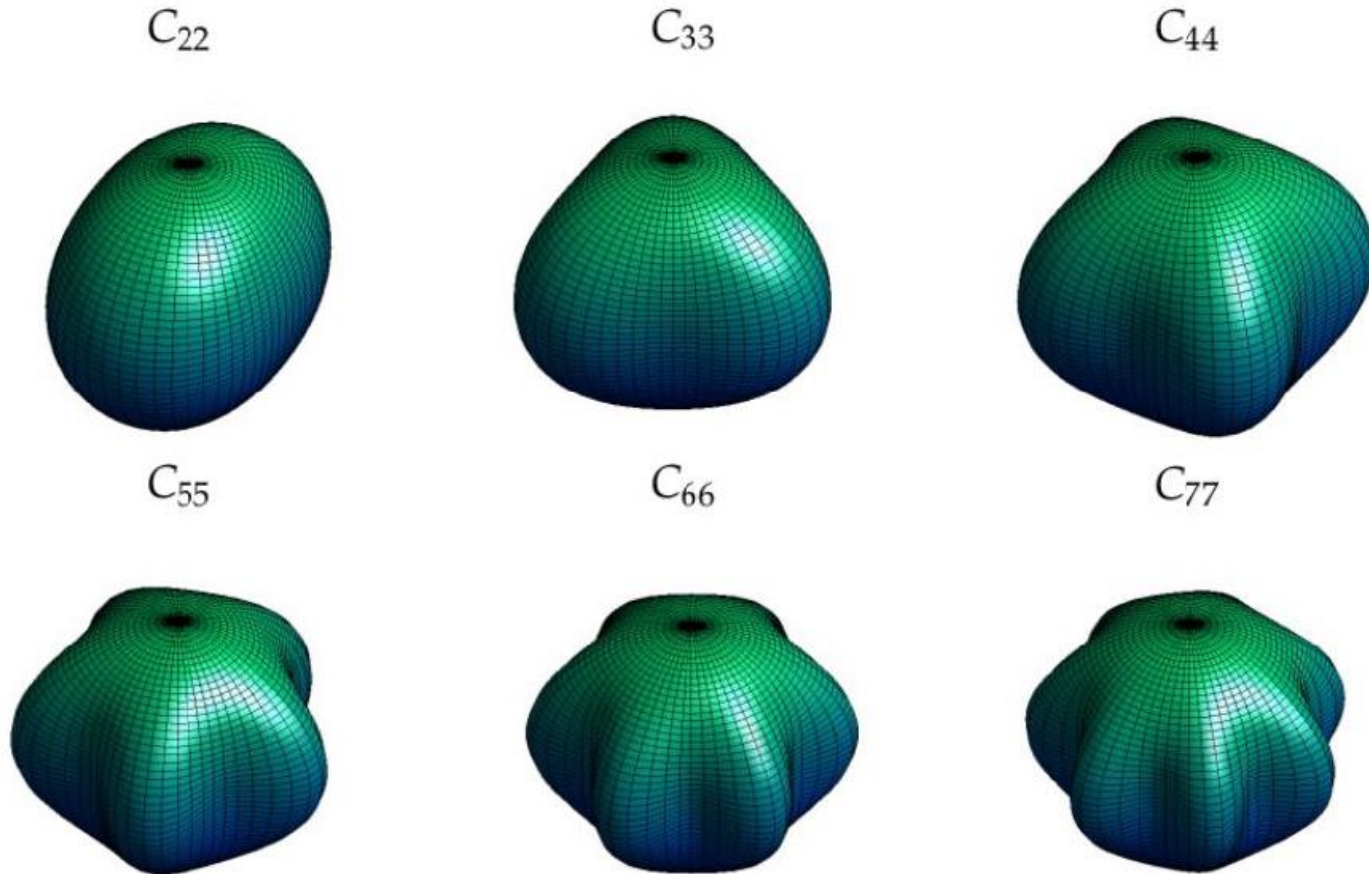
The sectorial coefficients represent bands of longitude.

The polynomials  $P_{l,l}$  are zero only at the poles.



Vallado, *Fundamental of Astrodynamics and Applications*, Kluwer, 2001.

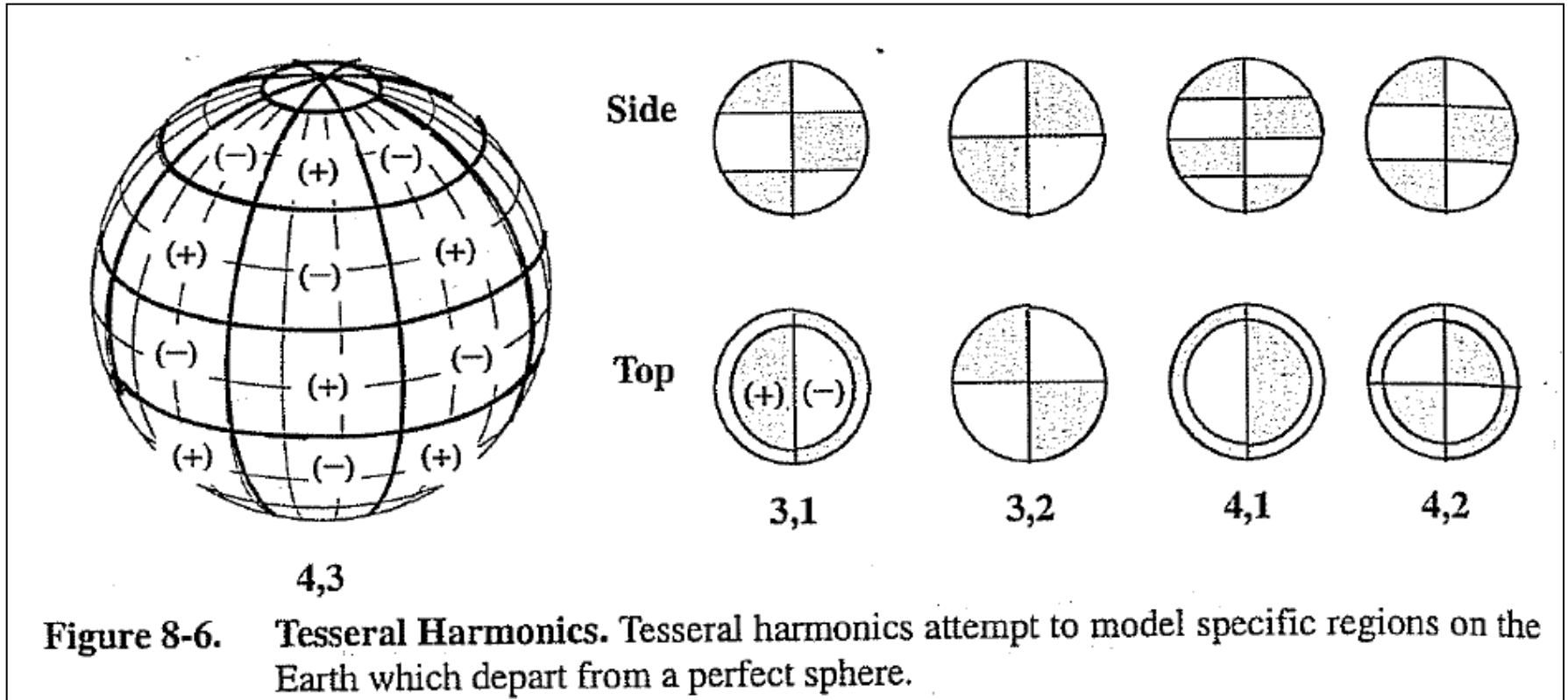
# Sectorial Harmonics ( $l=m$ )



Sectorial harmonics preserve symmetry with respect to the equatorial plane

Polynomials  $P_{l,l}$  are zero only at the poles

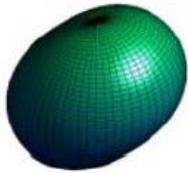
# Tesseral Harmonics ( $l \neq m \neq 0$ )



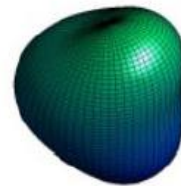
Vallado, *Fundamental of Astrodynamics and Applications*, Kluwer, 2001.

# Tesseral Harmonics ( $l \neq m \neq 0$ )

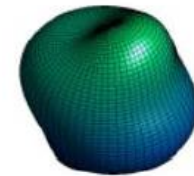
$C_{21}$



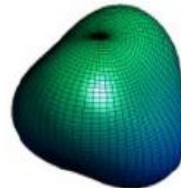
$C_{31}$



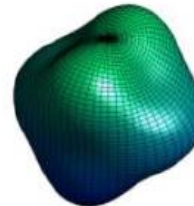
$C_{41}$



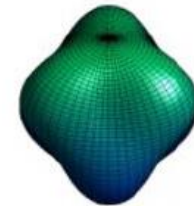
$C_{32}$



$C_{42}$



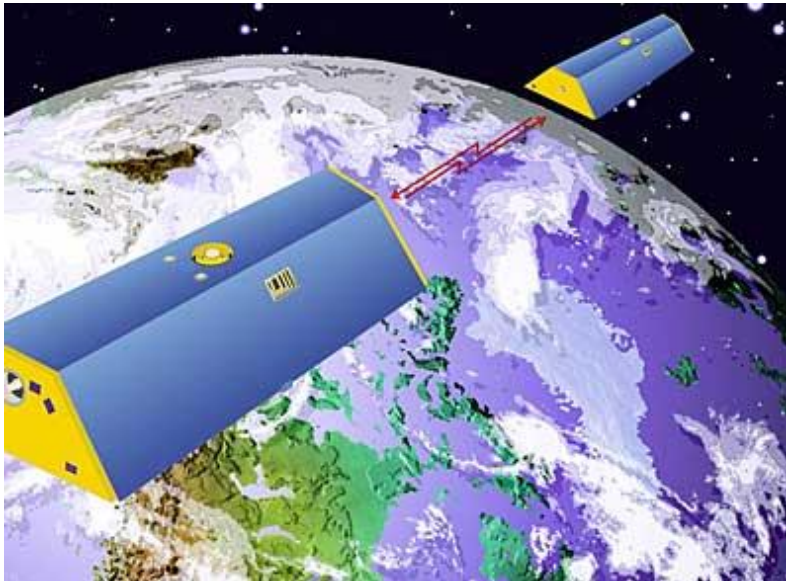
$C_{43}$



# Determination of Gravitational Coefficients

Because the internal distribution of the Earth is not known, the coefficients cannot be calculated from their definition.

They are determined experimentally; e.g, using satellite tracking.



Satellite-to-satellite tracking: GRACE employs microwave ranging system to measure changes in the distance between two identical satellites as they circle Earth. The ranging system detects changes as small as 10 microns over a distance of 220 km.

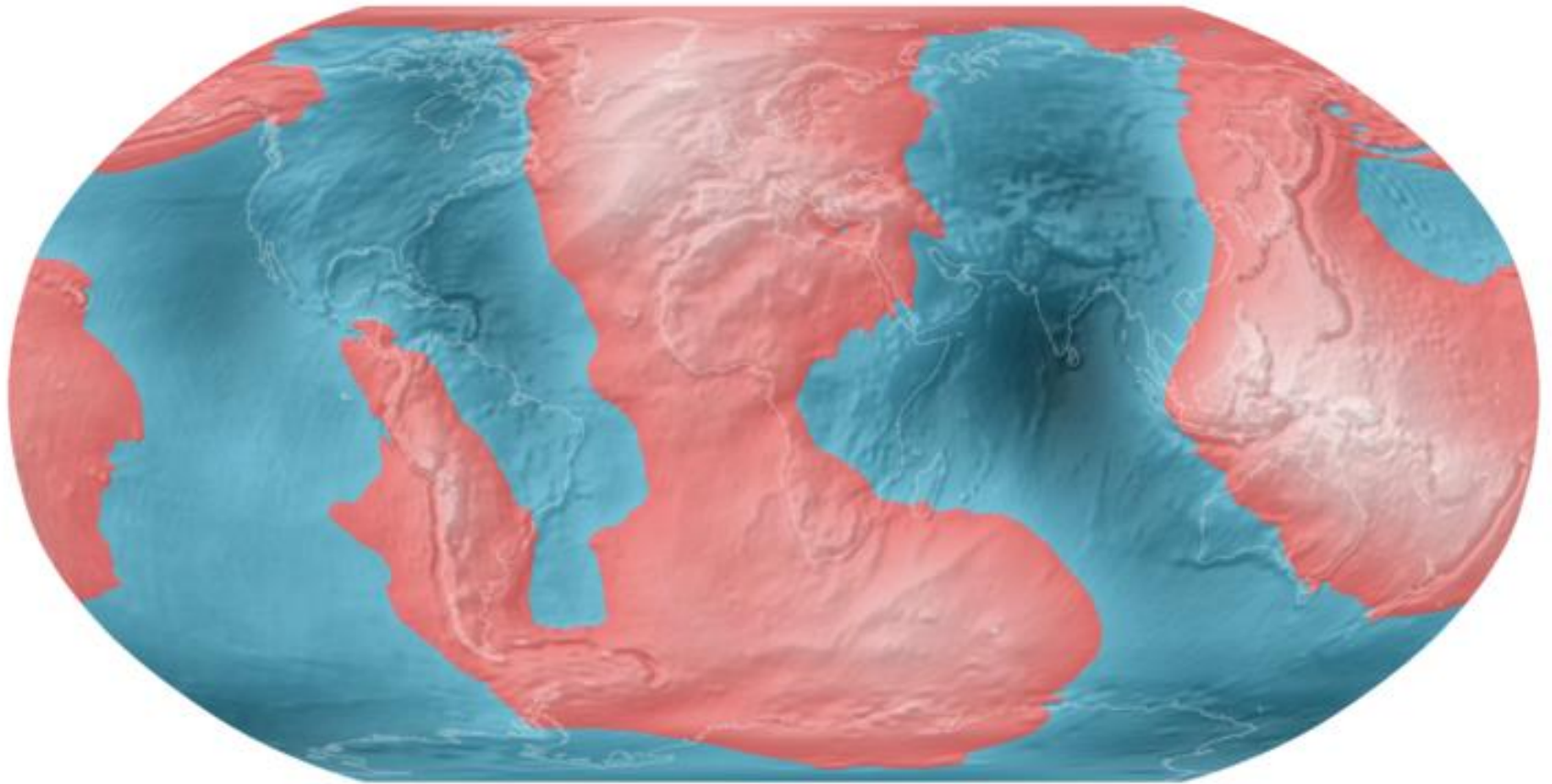
# Gravitational Coefficients: GRACE

EGM-2008 has been publicly released:

- ⇒ Extensive use of GRACE twin satellites.
- ⇒ 4.6 million terms in the spherical expansion (130317 in EGM-96)
- ⇒ Geoid with a resolution approaching 10 km (5'x5').

# Deviation of the Geoid from the idealized figure of the Earth

(difference between the EGM96 geoid and the WGS84 reference ellipsoid)

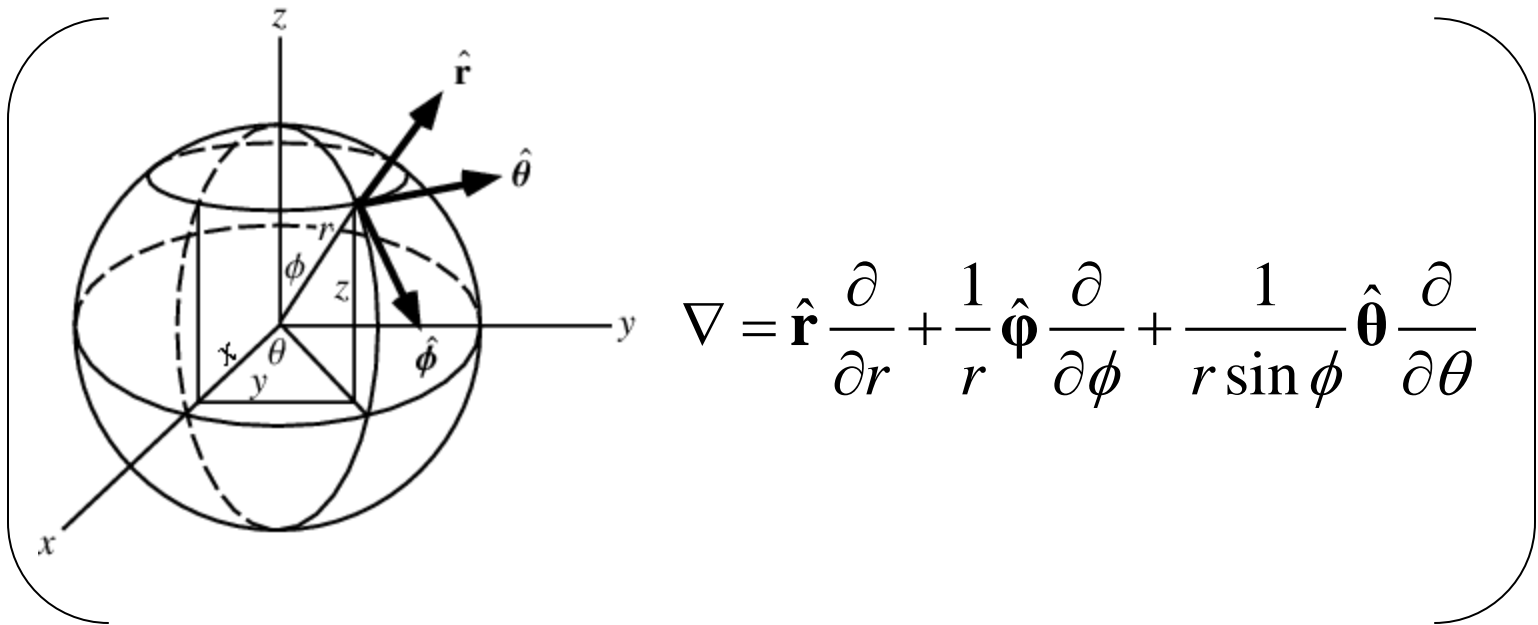


Red areas are above the idealized ellipsoid; blue areas are below.



# Resulting Force

$$\mathbf{F} = \nabla U \quad \text{with} \quad \nabla = \hat{\mathbf{r}} \frac{\partial}{\partial r} + \frac{1}{r} \hat{\boldsymbol{\phi}} \frac{\partial}{\partial \phi} + \frac{1}{r \cos \phi} \hat{\boldsymbol{\lambda}} \frac{\partial}{\partial \lambda}$$





# Spherical Earth

Gravitational force acts through the Earth's center.

$$U = \frac{\mu}{r}$$

$$\nabla = \hat{\mathbf{r}} \frac{\partial}{\partial r} + \frac{1}{r} \hat{\boldsymbol{\phi}} \frac{\partial}{\partial \phi} + \frac{1}{r \cos \phi} \hat{\boldsymbol{\lambda}} \frac{\partial}{\partial \lambda}$$

# Oblate Earth: J2

$$\left\{ \begin{aligned} U &= \frac{\mu}{r} \left\{ 1 + \sum_{l=2}^{\infty} \left( \frac{R_{\oplus}}{r} \right)^l \left[ -J_l P_l[\sin \phi_{sat}] + \sum_{m=1}^l \bar{P}_{lm}[\sin \phi_{sat}] \left[ \bar{C}_{l,m} \cos(m\lambda_{sat}) + \bar{S}_{l,m} \sin(m\lambda_{sat}) \right] \right] \right\} \\ P_2[\gamma] &= \frac{1}{2} (3\gamma^2 - 1) \end{aligned} \right\}$$

$$U = \frac{\mu}{r} \left\{ 1 - J_2 \left( \frac{R_{\oplus}}{r} \right)^2 \frac{3 \sin^2 \phi_{sat} - 1}{2} \right\}$$

$$\nabla = \hat{\mathbf{r}} \frac{\partial}{\partial r} + \frac{1}{r} \hat{\boldsymbol{\phi}} \frac{\partial}{\partial \phi} + \frac{1}{r \cos \phi} \hat{\boldsymbol{\lambda}} \frac{\partial}{\partial \lambda}$$

Perturbation of the radial acceleration

Longitudinal acceleration that can be decomposed into an azimuth and normal accelerations

# STK: Gravity Models (HPOP)

The screenshot displays the 'Force Model for ISS\_HPOP' dialog box in STK. The 'Central Body Gravity' section is active, with the 'Gravity' field set to 'WGS84\_EGM96.grv'. A red circle highlights this field. The 'Maximum Degree' and 'Maximum Order' are both set to 21. The 'Solid Tides' dropdown is set to 'Permanent tide only'. The 'Solar Radiation Pressure' section is also active, with 'Use' checked and 'Model' set to 'Spherical'. The 'Cr' field is set to 1.000000, and the 'Area/Mass Ratio' is set to 0.005 m<sup>2</sup>/kg. The 'Shadow Model' is set to 'Dual Cone'. The 'Third Body Gravity' section is visible at the bottom, showing a table of celestial bodies.

Name	Use	Source	Gravitational Constant
Sun	<input checked="" type="checkbox"/>	Cb file	1.32712200
Moon	<input checked="" type="checkbox"/>	Cb file	4.902801076000e+003 km <sup>3</sup> /sec <sup>2</sup>
Jupiter	<input type="checkbox"/>	Cb file	1.267127648383e+008 km <sup>3</sup> /sec <sup>2</sup>
Venus	<input type="checkbox"/>	Cb file	3.248585920790e+005 km <sup>3</sup> /sec <sup>2</sup>
Saturn	<input type="checkbox"/>	Cb file	3.794058536168e+007 km <sup>3</sup> /sec <sup>2</sup>

The 'Select a Gravity Field' dialog box is open, showing a file list in the 'Earth' directory. The file 'WGS84\_EGM96' is selected. The 'File name' field is empty, and the 'Files of type' dropdown is set to '\*.grv'. The 'Open as read-only' checkbox is unchecked.

# Atmospheric Drag

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Atmospheric forces represent the largest nonconservative perturbations acting on low-altitude satellites.

The drag is directly opposite to the velocity of the satellite, hence decelerating the satellite.

The lift force can be neglected in most cases.

# Mathematical Modeling

Knowledge of attitude      Velocity with respect to the atmosphere

$$\ddot{\mathbf{r}}_{\oplus sat} = -\frac{1}{2} C_D \frac{A}{m} \rho v_r^2 \frac{\mathbf{v}_r}{v_r}$$

[1.5-3]      Atmospheric density

The diagram shows the drag force equation  $\ddot{\mathbf{r}}_{\oplus sat} = -\frac{1}{2} C_D \frac{A}{m} \rho v_r^2 \frac{\mathbf{v}_r}{v_r}$  enclosed in a grey box. Four arrows point from the box to labels: 'Knowledge of attitude' points to the attitude matrix  $\mathbf{C}_D$ ; 'Velocity with respect to the atmosphere' points to the relative velocity vector  $\mathbf{v}_r$ ; '[1.5-3]' points to the drag coefficient  $C_D$ ; and 'Atmospheric density' points to the density  $\rho$ .

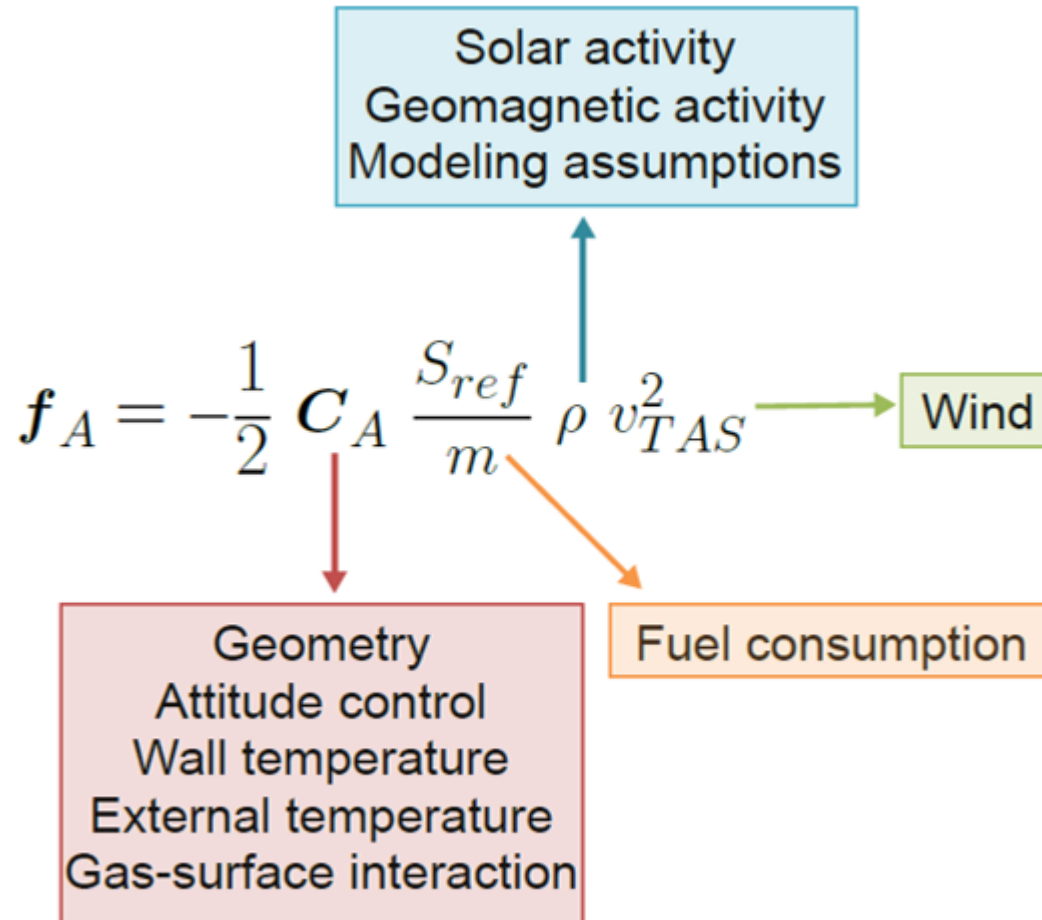
The atmosphere co-rotates with the Earth.

$$\mathbf{v}_r = \mathbf{v} - \boldsymbol{\omega}_{\oplus} \times \mathbf{r}$$

Inertial velocity      Earth's angular velocity

The diagram shows the equation  $\mathbf{v}_r = \mathbf{v} - \boldsymbol{\omega}_{\oplus} \times \mathbf{r}$ . Two arrows point from the terms  $\mathbf{v}$  and  $\boldsymbol{\omega}_{\oplus}$  to the labels 'Inertial velocity' and 'Earth's angular velocity' respectively.

# Aerodynamic Drag Force Terms Are Uncertain



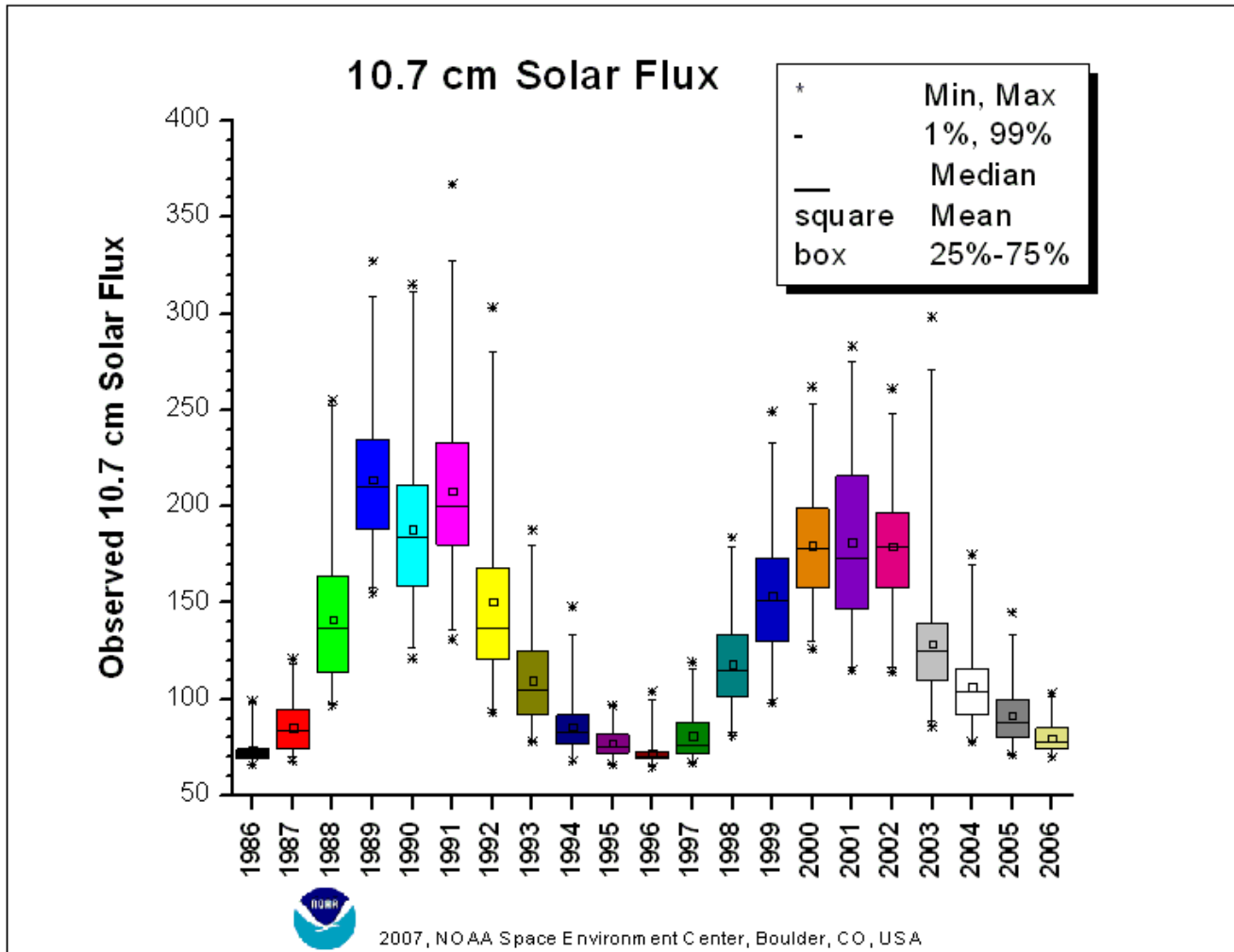
# Atmospheric Density

The gross behavior of the atmospheric density is well established, but it is still this factor which makes the determination of satellite lifetimes so uncertain.

There exist several models (e.g., Jacchia-Roberts, Harris-Priester).

Dependence on temperature, molecular weight, altitude, solar activity, etc.

# Solar Activity





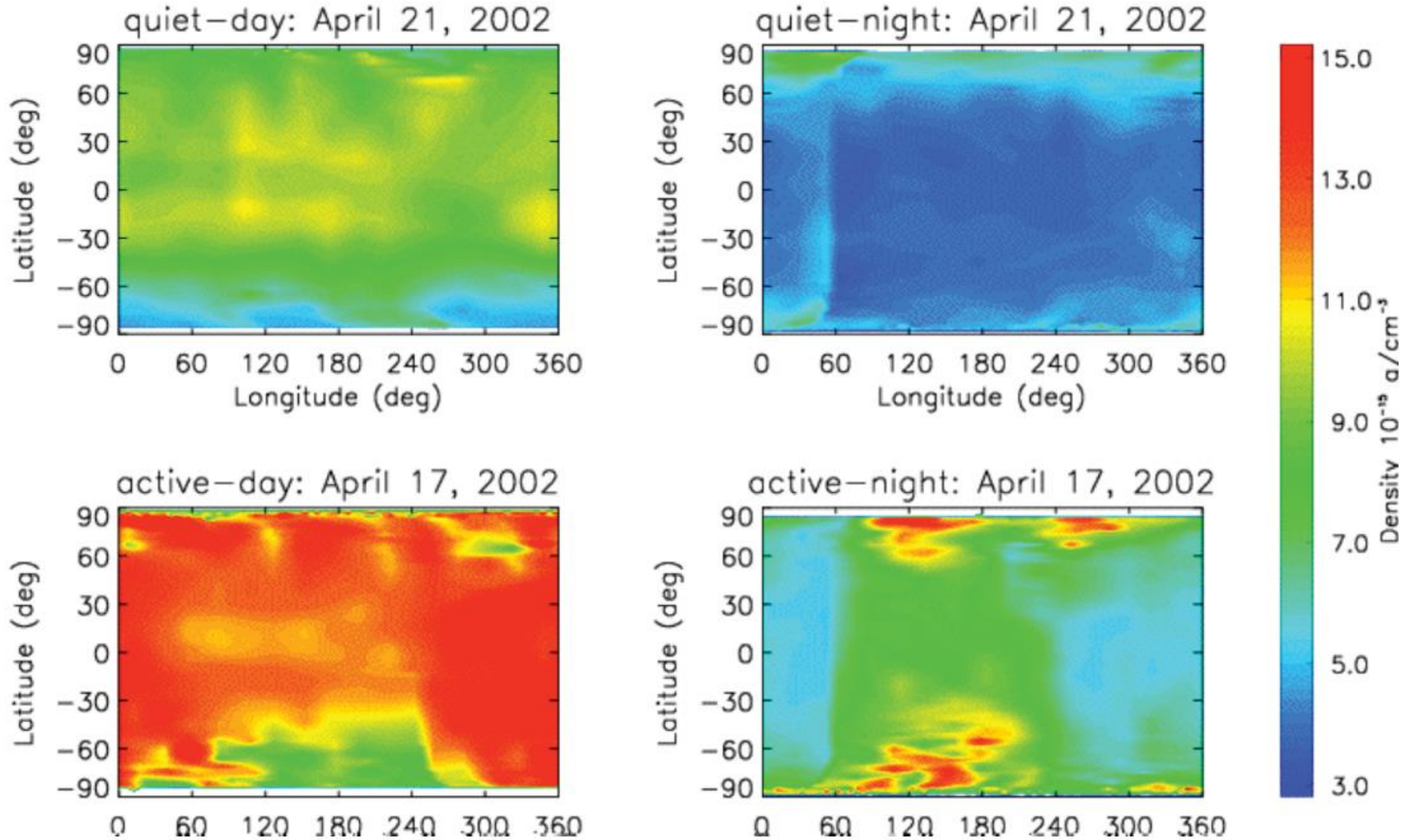
# Atmospheric Density using CHAMP



An accelerometer measures the non-gravitational accelerations in three components, of which the along-track component mainly represents the atmospheric drag.

By subtracting modeled accelerations for SRP and Earth Albedo, the drag acceleration is isolated and is proportional to the atmospheric density.

# CHAMP Density at 410 kms



# Further Reading



Available online at [www.sciencedirect.com](http://www.sciencedirect.com)



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**Planetary  
and  
Space Science**

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[www.elsevier.com/locate/pss](http://www.elsevier.com/locate/pss)

## Atmospheric densities derived from CHAMP/STAR accelerometer observations

S. Bruinsma\*, D. Tamagnan, R. Biancale

*CNES, Department of Terrestrial and Planetary Geodesy, 18, Avenue E. Belin, Toulouse 31401, Cedex 4, France*

Received 29 January 2003; received in revised form 2 October 2003; accepted 21 October 2003

# Harris-Priester (120-2000km)

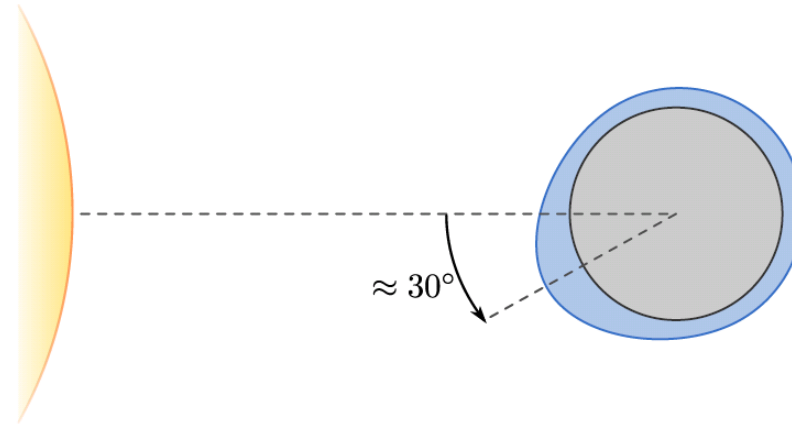
---

Static model (e.g., no variation with the 27-day solar rotation).

Interpolation determines the density at a particular time.

Simple, computationally efficient and fairly accurate.

# Atmospheric Bulge



The high atmosphere bulges toward a point in the sky some  $15^\circ$  to  $30^\circ$  east of the sun (density peak at 2pm local solar time).

The observed accelerations of Vanguard satellite (1958) indicated that the air density at 665 km is about 10 times as great when perigee passage occur one hour after noon as when it occurs during the night !

# Harris-Priester (120-2000km)

The modified Harris-Priester (HP) model may be considered as a middle ground between the two extremes (Harris and Priester, 1962; Long et al., 1989; Hatten and Russell, 2016). Like the Standard Atmosphere, HP relies on exponential interpolation of density between values tabulated at discrete altitudes. However, HP also uses functional dependencies to model latitudinal and diurnal effects. Further, HP may be revised to take into account varying levels of solar activity. This effect has been achieved by including a set of 10 tables, each of which corresponds to a different value of the 81-day centered average 10.7 cm solar flux index  $\bar{F}_{10.7}$ . Given a value of  $\bar{F}_{10.7}$ , an interpolation scheme, such as nearest-neighbor (Dowd and Tapley, 1979) or linear (Tolman et al., 2004), is used to calculate density values.

Thus, HP may produce significantly more accurate density values than a simple exponential atmospheric model while executing in a fraction of the time of more complex models (Montenbruck and Gill, 2001). Such balance makes HP a suitable candidate for use in preliminary studies in which a combination of high speed and reasonable accuracy is paramount. However, even in this context, the HP model is not without its deficiencies. This work addresses

# Harris-Priester (120-2000km)

Account for diurnal density bulge due to solar radiation

$$\rho(h) = \rho_m(h) + [\rho_M(h) - \rho_m(h)] \cos^n \left( \frac{\psi}{2} \right)$$

Height above the Earth's  
reference ellipsoid

Angle between  
satellite position  
vector and the apex  
of the diurnal bulge

# Interpolation Between Altitudes

$$\rho_m(h) = \rho_m(h_i) \exp\left(\frac{h_i - h}{H_{m_i}}\right), \quad h_i \leq h \leq h_{i+1}$$

$$\rho_M(h) = \rho_M(h_i) \exp\left(\frac{h_i - h}{H_{M_i}}\right), \quad h_i \leq h \leq h_{i+1},$$

$$H_{m_i} = \frac{h_i - h_{i+1}}{\ln\left(\frac{\rho_m(h_{i+1})}{\rho_m(h_i)}\right)}$$

$$H_{M_i} = \frac{h_i - h_{i+1}}{\ln\left(\frac{\rho_M(h_{i+1})}{\rho_M(h_i)}\right)}$$

$h$ [km]	$\rho_m$ [g/km <sup>3</sup> ]	$\rho_M$ [g/km <sup>3</sup> ]	$h$ [km]	$\rho_m$ [g/km <sup>3</sup> ]	$\rho_M$ [g/km <sup>3</sup> ]
100	497400.0	497400.0	420	1.558	5.684
120	24900.0	24900.0	440	1.091	4.355
130	8377.0	8710.0	460	0.7701	3.362
140	3899.0	4059.0	480	0.5474	2.612
150	2122.0	2215.0	500	0.3916	2.042
160	1263.0	1344.0	520	0.2819	1.605
170	800.8	875.8	540	0.2042	1.267
180	528.3	601.0	560	0.1488	1.005
190	361.7	429.7	580	0.1092	0.7997
200	255.7	316.2	600	0.08070	0.6390
210	183.9	239.6	620	0.06012	0.5123
220	134.1	185.3	640	0.04519	0.4121
230	99.49	145.5	660	0.03430	0.3325
240	74.88	115.7	680	0.02632	0.2691
250	57.09	93.08	700	0.02043	0.2185
260	44.03	75.55	720	0.01607	0.1779
270	34.30	61.82	740	0.01281	0.1452
280	26.97	50.95	760	0.01036	0.1190
290	21.39	42.26	780	0.008496	0.09776
300	17.08	35.26	800	0.007069	0.08059
320	10.99	25.11	840	0.004680	0.05741
340	7.214	18.19	880	0.003200	0.04210
360	4.824	13.37	920	0.002210	0.03130
380	3.274	9.955	960	0.001560	0.02360
400	2.249	7.492	1000	0.001150	0.01810

Mean solar activity



# Atmospheric Bulge Position

Spacecraft position (ECI)  $\swarrow$   $\searrow$  Unit vector toward the apex of the diurnal bulge in ECI coordinates

$$\cos^n \left( \frac{\psi}{2} \right) = \left( \frac{1}{2} + \frac{\mathbf{r}^T \mathbf{u}_b}{2r} \right)^{\frac{n}{2}}$$

$$\mathbf{u}_b = \begin{pmatrix} \cos(\delta_s) \cos(\alpha_s + \lambda_{lag}) \\ \cos(\delta_s) \sin(\alpha_s + \lambda_{lag}) \\ \sin(\delta_s) \end{pmatrix} \rightarrow \text{Lag}$$

$\swarrow$  Sun declination  $\searrow$  Sun right ascension

# STK – Atmospheric Models (HPOP)

The screenshot displays the 'Force Model for ISS' dialog box in the STK software. The 'Atm. Density Model' dropdown menu is open, showing a list of models. 'Jacchia-Roberts' is selected and highlighted in blue. A red circle is drawn around the dropdown menu.

**Central Body Gravity**

Gravity: WGS84\_EGM96.grv  
Maximum Degree: 21  
Maximum Order: 21  
Solid Tides: Permanent tide only  
 Use Ocean Tides

**Solar Radiation Pressure**

Use  
Cr: 1.000000  
Area/Mass Ratio: 0.02 m<sup>2</sup>/kg  
Shadow Model: Dual Cone  
 Use Boundary Mitigation

**Drag**

Use  
Cd: 2.200000  
Area/Mass Ratio: 0.02 m<sup>2</sup>/kg  
Atm. Density Model: Jacchia-Roberts  
SolarFlux/GeoMag: 1976 Standard, Harris-Priester, Jacchia 1970, Jacchia 1971, NRLMSISE 2000, MSISE 1990, MSIS 1986, Jacchia 1960, Jacchia-Roberts, CIRA 1972  
Daily F10.7:  
Average F10.7:  
Geomagnetic Index (Kp):

**Third Body Gravity**

Name	Use	Source	Gravity Value
Sun	<input checked="" type="checkbox"/>	Cb file	1.327122000000e+011 km <sup>3</sup> /sec <sup>2</sup>
Moon	<input checked="" type="checkbox"/>	Cb file	4.902802953597e+003 km <sup>3</sup> /sec <sup>2</sup>
Jupiter	<input type="checkbox"/>	Cb file	1.267127678578e+008 km <sup>3</sup> /sec <sup>2</sup>
Venus	<input type="checkbox"/>	Cb file	3.248585920790e+005 km <sup>3</sup> /sec <sup>2</sup>
Saturn	<input type="checkbox"/>	Cb file	3.794062606114e+007 km <sup>3</sup> /sec <sup>2</sup>

More Options...  
OK Cancel Help

# STK – Solar Activity (HPOP)

The screenshot displays the 'Force Model for ISS\_HPOP' dialog box in the STK software. The interface is organized into several sections:

- Central Body Gravity:** Gravity is set to 'WGS84\_EGM96.grv'. Maximum Degree and Maximum Order are both set to 21. Solid Tides is set to 'Permanent tide only'. The 'Use Ocean Tides' checkbox is unchecked.
- Drag:** The 'Use' checkbox is checked. Cd is set to 2.070000. Area/Mass Ratio is set to 0.005 m<sup>2</sup>/kg. Atm. Density Model is set to 'Harris-Priester'.
- Solar Radiation Pressure:** The 'Use' checkbox is checked. The Model is set to 'Spherical'. Cr is set to 1.000000. Area/Mass Ratio is set to 0.005 m<sup>2</sup>/kg. Shadow Model is set to 'Dual Cone'. The 'Use Boundary Mitigation' checkbox is unchecked.
- SolarFlux/GeoMag:** This section is circled in red. It includes a dropdown menu set to 'Enter Manually'. The 'Daily F10.7' field is set to 150.00000000. The 'Average F10.7' field is set to 150.00000000. The 'Geomagnetic Index (Kp)' field is set to 3.00000000.

At the bottom of the dialog, there is a button labeled 'Eclipsing Bodies...'. On the left side of the image, a tree view shows the 'Basic' tab selected, with 'Orbit' highlighted.

# Third-Body Perturbations



For an Earth-orbiting satellite, the Sun and the Moon should be modeled for accurate predictions.

Their effects become noticeable when the effects of drag begin to diminish.

# STK: Third-Body Gravity (HPOP)

Scenario1 - STK 8 - [OUFTI-1 : Basic Orbit]

File Edit View Insert Tools Satellite Window Feedback Help

Object Browser

Scenario1  
OUFTI-1

Basic

Propagator: HPOP

Start Time: 1 Jul 2007 12:00:00.000 UTCG

Stop Time: 2 Jul 2007 12:00:00.000 UTCG

Step Size: 60 sec

Orbit Epoch: 1 Jul 2007 12:00:00.000 UTCG

Coord Epoch: 1 Jan 2000 11:58:55.816 UTCG

Coord Type: Classical

Coord System: J2000

Prop Specific: Force Models...  
Integrator...  
Covariance...

Force Model for OUFTI-1

Central Body Gravity

Gravity: WGS84\_EGM96.grv

Maximum Degree: 21

Maximum Order: 21

Solid Tides: Permanent tide only

Use Ocean Tides

Solar Radiation Pressure

Use

Cr: 1.000000

Area/Mass Ratio: 0.02 m<sup>2</sup>/kg

Shadow Model: Dual Cone

Use Boundary Mitigation

Drag

Use

Cd: 2.200000

Area/Mass Ratio: 0.02 m<sup>2</sup>/kg

Atm. Density Model: Jacchia-Roberts

Solar Flux/GeoMag

Enter Manually

Daily F10.7: 150.00000000

Average F10.7: 150.00000000

Geomagnetic Index (Kp): 3.00000000

Third Body Gravity

Name	Use	Source	Gravity Value
Sun	<input checked="" type="checkbox"/>	Cb file	1.327122000000e+011 km <sup>3</sup> /sec <sup>2</sup>
Moon	<input checked="" type="checkbox"/>	Cb file	4.902802953597e+003 km <sup>3</sup> /sec <sup>2</sup>
Jupiter	<input type="checkbox"/>	Cb file	1.267127678578e+008 km <sup>3</sup> /sec <sup>2</sup>
Venus	<input type="checkbox"/>	Cb file	3.248585920790e+005 km <sup>3</sup> /sec <sup>2</sup>
Saturn	<input type="checkbox"/>	Cb file	3.794062606114e+007 km <sup>3</sup> /sec <sup>2</sup>

More Options...

OK Cancel Apply Help

LaunchPad 3D Graphic... 2D Graphic... OUFTI-1 : B...

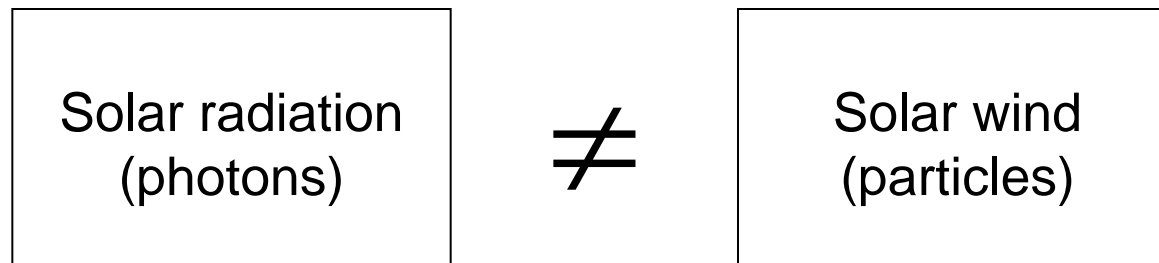
OUFTI-1 (23.119,-104.862) 1 Jul 2007 12:00:00.000 Time Step: 60.00 sec

# Solar Radiation Pressure

It produces a nonconservative perturbation on the spacecraft, which depends upon the distance from the sun.

It is usually very difficult to determine precisely.

It is NOT related to solar wind, which is a continuous stream of particles emanating from the sun.



800km is regarded as a transition altitude between drag and SRP.

# Mathematical Modeling

$$\mathbf{F}_{SR} = -p_{SR} c_R A \mathbf{e}_{sc / sun}$$

$$p_{SR} = \frac{1350 \text{ W/m}^2}{3e8 \text{ m/s}} = 4.51 \times 10^{-6} \text{ N/m}^2$$

The reflectivity  $c_R$  is a value between 0 and 2:

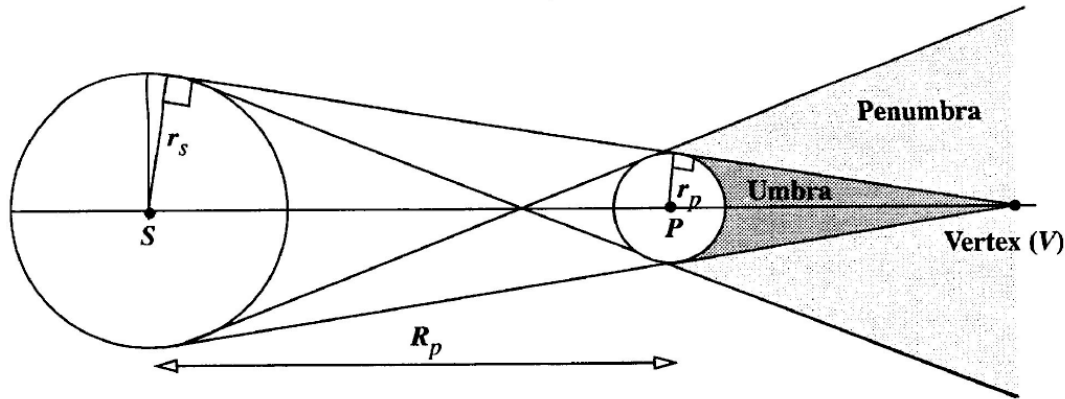
- 0: translucent to incoming radiation.
- 1: all radiation is absorbed (black body).
- 2: all radiation is reflected.

The incident area exposed to the sun must be known. The normals to the surfaces are assumed to point in the direction of the sun (e.g., solar arrays).

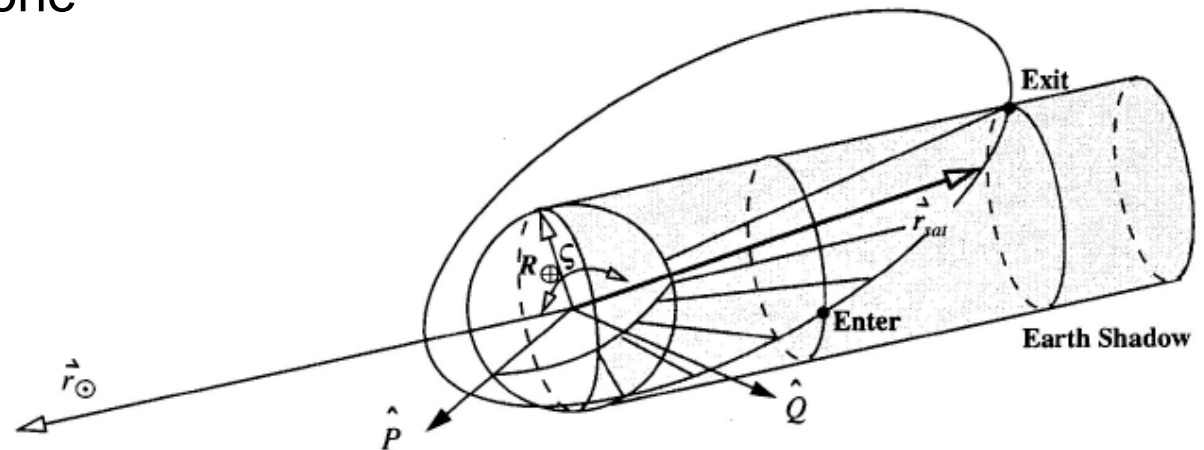
# Mathematical Modeling: Eclipses

Use of shadow functions:

$$\mathbf{F}_{SR} = 0$$



Dual cone



Cylindrical



# STK: Solar Radiation Pressure (HPOP)

The screenshot shows the STK software interface with the 'Force Model for OUFTI-1' dialog box open. The 'Solar Radiation Pressure' section is highlighted with a red circle. The dialog box contains the following settings:

- Central Body Gravity:**
  - Gravity: WGS84\_EGM96.grv
  - Maximum Degree: 21
  - Maximum Order: 21
  - Solid Tides: Permanent tide only
  - Use Ocean Tides
- Drag:**
  - Use
  - Cd: 2.200000
  - Area/Mass Ratio: 0.02 m<sup>2</sup>/kg
  - Atm. Density Model: Jacchia-Roberts
- Solar Flux/GeoMag:**
  - Enter Manually
  - Daily F10.7: 150.00000000
  - Average F10.7: 150.00000000
  - Geomagnetic Index (Kp): 3.00000000
- Solar Radiation Pressure (highlighted):**
  - Use
  - Cr: 1.000000
  - Area/Mass Ratio: 0.02 m<sup>2</sup>/kg
  - Shadow Model: Dual Cone
  - Use Boundary Mitigation
- Third Body Gravity:**

Name	Use	Source	Gravity Value
Sun	<input checked="" type="checkbox"/>	Cb file	1.327122000000e+011 km <sup>3</sup> /sec <sup>2</sup>
Moon	<input checked="" type="checkbox"/>	Cb file	4.902802953597e+003 km <sup>3</sup> /sec <sup>2</sup>
Jupiter	<input type="checkbox"/>	Cb file	1.267127678578e+008 km <sup>3</sup> /sec <sup>2</sup>
Venus	<input type="checkbox"/>	Cb file	3.248585920790e+005 km <sup>3</sup> /sec <sup>2</sup>
Saturn	<input type="checkbox"/>	Cb file	3.794062606114e+007 km <sup>3</sup> /sec <sup>2</sup>

The 'Propagator' is set to HPOP. The 'Start Time' is 1 Jul 2007 12:00:00.000 UTCG, and the 'Stop Time' is 2 Jul 2007 12:00:00.000 UTCG. The 'Step Size' is 60 sec. The 'Orbit Epoch' is 1 Jul 2007 12:00:00.000 UTCG, and the 'Coord Epoch' is 1 Jan 2000 11:58:55.816 UTCG. The 'Coord Type' is Classical, and the 'Coord System' is J2000. The 'Prop Specific' buttons are Force Models..., Integrator..., and Covariance... The 'More Options...' button is also visible.

# STK: Shadow Models (HPOP)

**Force Model for Satellite1**

Central Body Gravity

Gravity: WGS84\_EGM96.grv ...

Maximum Degree: 21

Maximum Order: 21

Solid Tides: Permanent tide only

Use Ocean Tides

Solar Radiation Pressure

Use

Cr: 1.000000

Area/Mass Ratio: 0.02 m<sup>2</sup>/kg

Shadow Model: Dual Cone

Use Boundary Model

Drag

Use

Cd: 2.200000

Area/Mass Ratio: 0.02 m<sup>2</sup>/kg

Atm. Density Model: Jacchia-Roberts

SolarFlux/GeoMag

Enter Manually

Daily F10.7: 150.00000000

Average F10.7: 150.00000000

Geomagnetic Index (Kp): 3.00000000

Third Body Gravity

Name	Use	Source	Gravity Value
Sun	<input checked="" type="checkbox"/>	Cb file	1.327122000000e+011 km <sup>3</sup> /sec <sup>2</sup>
Moon	<input checked="" type="checkbox"/>	Cb file	4.902802953597e+003 km <sup>3</sup> /sec <sup>2</sup>
Jupiter	<input type="checkbox"/>	Cb file	1.267127678578e+008 km <sup>3</sup> /sec <sup>2</sup>
Venus	<input type="checkbox"/>	Cb file	3.248585920790e+005 km <sup>3</sup> /sec <sup>2</sup>
Saturn	<input type="checkbox"/>	Cb file	3.794062608114e+007 km <sup>3</sup> /sec <sup>2</sup>

More Options...

OK Cancel Help

# STK: Central Body Pressure (HPOP)

Scenario 1 - STK 8 - [Satellite1 : Basic Orbit]

File Edit View Insert Tools Satellite Window Feedback Help

Object Browser

- Scenario1
  - Satellite1

### Force Model for Satellite 1

Central Body Gravity

Gravity: WGS84\_EGM96.grv ...

Maximum Degree: 21

Maximum Order: 21

Solid Tides: Permanent tide only

Use Ocean Tides

Solar Radiation Pressure

Use

Cr: 1.000000

Area/Mass Ratio: 0.02 m<sup>2</sup>/kg

Shadow Model: Dual Cone

Use Boundary Mitigation

Third Body Gravity

Name	Use	Source	Gravity Value
Pluto	<input type="checkbox"/>	Cb file	1.0090760000000e+003 km <sup>3</sup> /sec <sup>2</sup>
Charon	<input type="checkbox"/>	Cb file	1.0810260000000e+002 km <sup>3</sup> /sec <sup>2</sup>
Phobos	<input type="checkbox"/>	Cb file	7.0933990000000e-004 km <sup>3</sup> /sec <sup>2</sup>
Deimos	<input type="checkbox"/>	Cb file	1.5881740000000e-004 km <sup>3</sup> /sec <sup>2</sup>

Drag

Cd:

Area/Mass Ratio:

Atm. Density Model:

SolarFlux/GeoMag

Daily F10.7:

Average F10.7:

Geomagnetic Index (Kp):

More Options...

OK

### Force Model Options - Satellite 1

Drag

Use Approximate Altitude

Use Apparent Sun Position

Satellite Mass: 1000 kg

Include Relativistic Accelerations

Solar Radiation Pressure

Method to Compute Sun Position: Apparent To True CB

Atmospheric Altitude for the shape of the central body for Eclipse: 0 km

Solid Tides

Include Time Dependent Solid Tides

Minimum Amplitude: 0 m

Ocean Tides

Maximum Degree: 4

Maximum Order: 4

Minimum Amplitude: 0 m

Propagator Plugin

Use Plugin

Plugin Name:

Central Body Radiation Pressure

Include Albedo Ck: 1.000000

Include Thermal Area to Mass Ratio: 0.02 m<sup>2</sup>/k

Ground Reflection Model File: SimpleReflectionModel.txt

OK Cancel Help

# S3L Propagator

### Keplerian Parameters

Semi-major axis [m]	<input type="text" value="6778e3"/>
Eccentricity	<input type="text" value="0.0"/>
Inclination [deg]	<input type="text" value="51"/>
Argument of perigee [deg]	<input type="text" value="0.0"/>
RAAN [deg]	<input type="text" value="20"/>
True anomaly [deg]	<input type="text" value="0.0"/>

### Control

None  
 Cross-Section  
 Attitude

### Date

Year	<input type="text" value="2010"/>
Month	<input type="text" value="10"/>
Day	<input type="text" value="23"/>
Hours	<input type="text" value="19"/>
Minutes	<input type="text" value="40"/>
Seconds	<input type="text" value="00"/>
Simulation time [s]	<input type="text" value="5*24 * 3600"/>

### Force Model

Non-spherical  
 Drag  
 SRP  
 Third-body Sun  
 Third-body Sun

### ECI to ECEF

Precession  
 Nutation  
 Polar Wandering

### Integration Parameters

Relative tolerance	<input type="text" value="1e-13"/>
Absolute tolerance	<input type="text" value="1e-13"/>
Output time step [s]	<input type="text" value="60"/>

### Spacecraft Properties

Mass [kg]	<input type="text" value="4"/>
Sizes [m, m, m]	<input type="text" value="[0.3, 0.1, 0.1]"/>
Cross-section to TAS [m^2]	<input type="text" value="0.03"/>
Cross-section to Sun [m^2]	<input type="text" value="0.03"/>
Drag Coefficient	<input type="text" value="4"/>
Reflectivity Coefficient	<input type="text" value="[1.2, 1.2, 1.2, 1.2, 1.2, 1.2]"/>

### Density Model

Harris-Priester  
 Jacchia 71  
 Jacchia-Roberts  
 Measured data

### Density Parameters

Harris-Priester coeff.	<input type="text" value="0"/>
DailyF10.7	<input type="text" value="155"/>
Averaged F10.7	<input type="text" value="155"/>
Geomagnetic activity	<input type="text" value="3"/>

### Gravity Model

Maximum Degree	<input type="text" value="2"/>
Maximum Order	<input type="text" value="0"/>

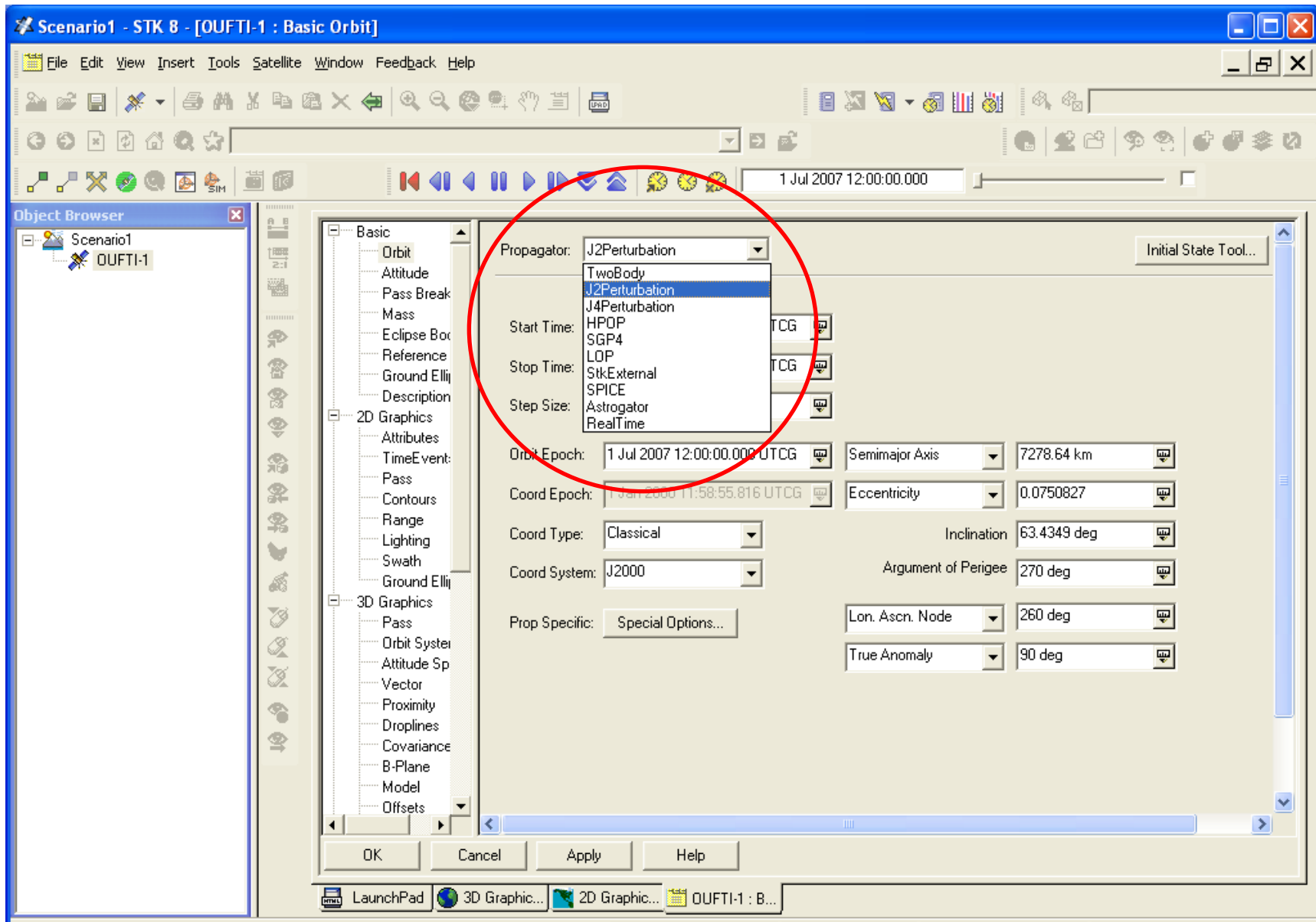
Download Data



Orbit 3D

**RUN!**

# STK: Different Propagators





# What is the Highest Point on Earth ?

# What is the Highest Point on Earth ?

Mount Chimborazo (6310 m), located in Ecuador, may be considered as the highest point on Earth. It is the spot on the surface farthest from the Earth's center.

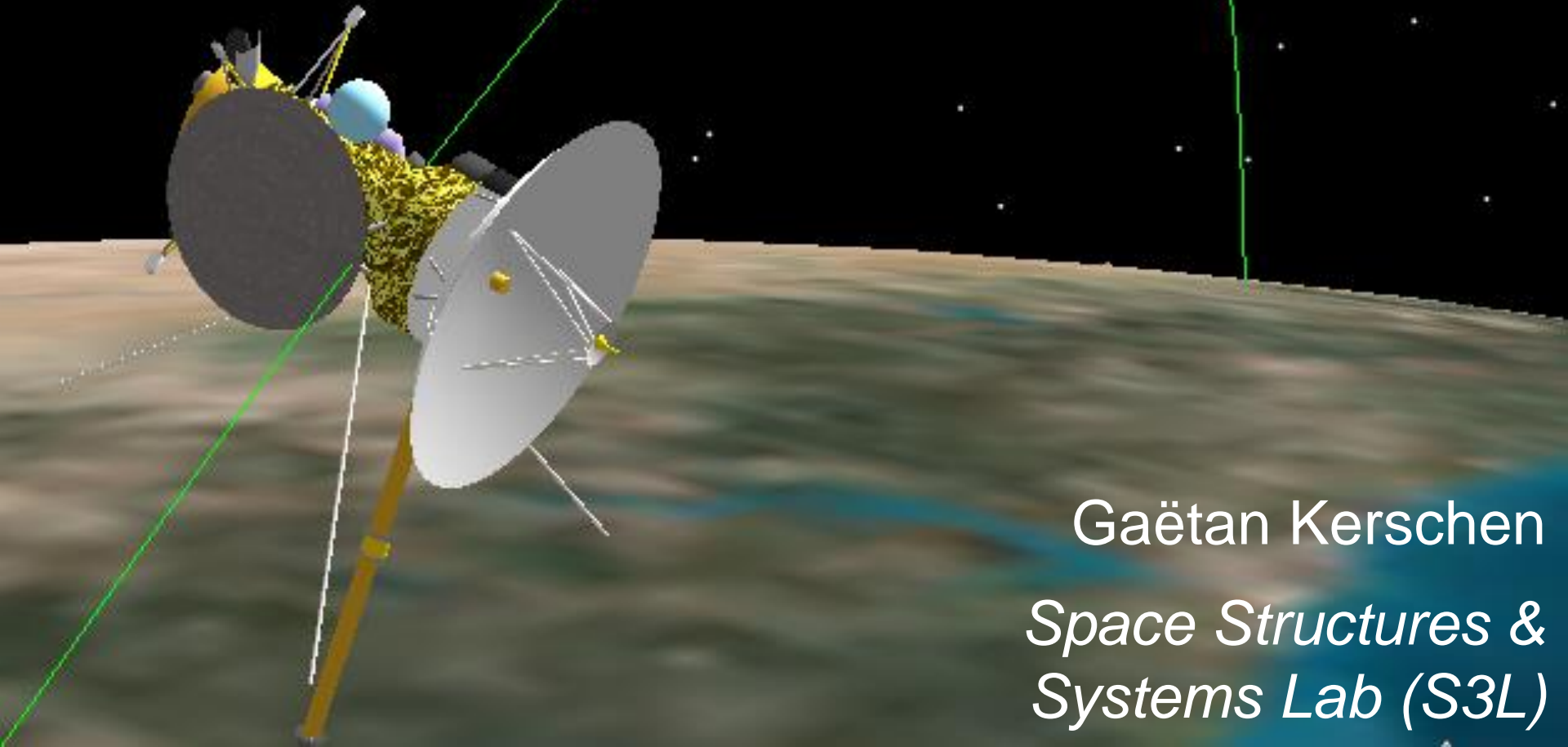


*6384.4 km (Chimborazo) vs. 6382.3 km (Everest)*

Cassini Classical Orbit Elements  
Time (UTCG): 15 Oct 1997 09:18:54.000  
Semi-major Axis (km): 6685.637000  
Eccentricity: 0.020566  
Inclination (deg): 30.000  
RAAN (deg): 150.546  
Arg of Perigee (deg): 230.000  
True Anomaly (deg): 136.530  
Mean Anomaly (deg): 134.891

# Aerodynamics (AERO0024)

## 5. Dominant Perturbations



Gaëtan Kerschen  
*Space Structures &  
Systems Lab (S3L)*