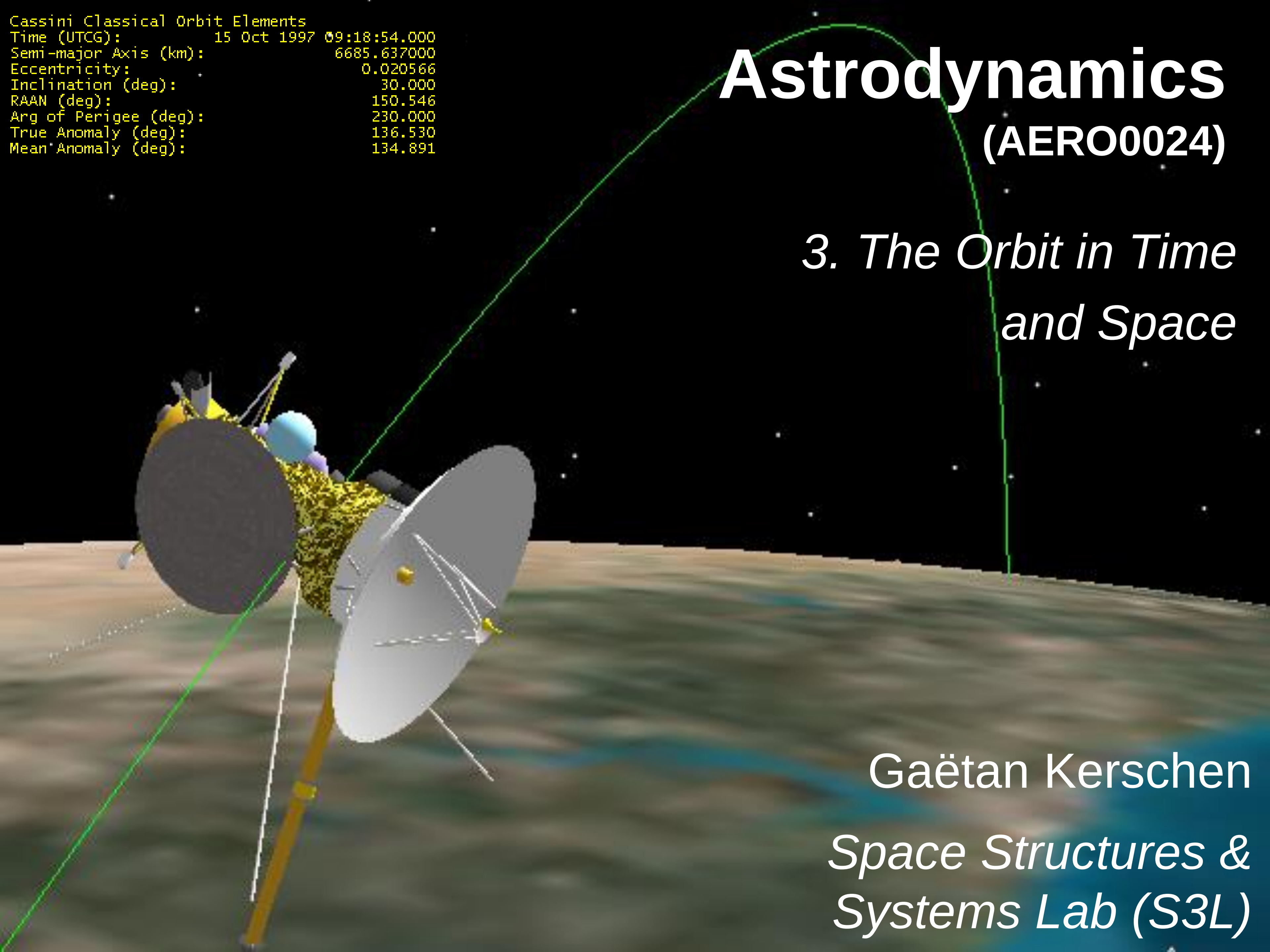


Astrodynamics
(AERO0024)

3. The Orbit in Time and Space

Gaëtan Kerschen

Space Structures & Systems Lab (S3L)

Importance/Complexity of Time Measurement

Applications such as GPS rely on an extremely precise time measurement system:

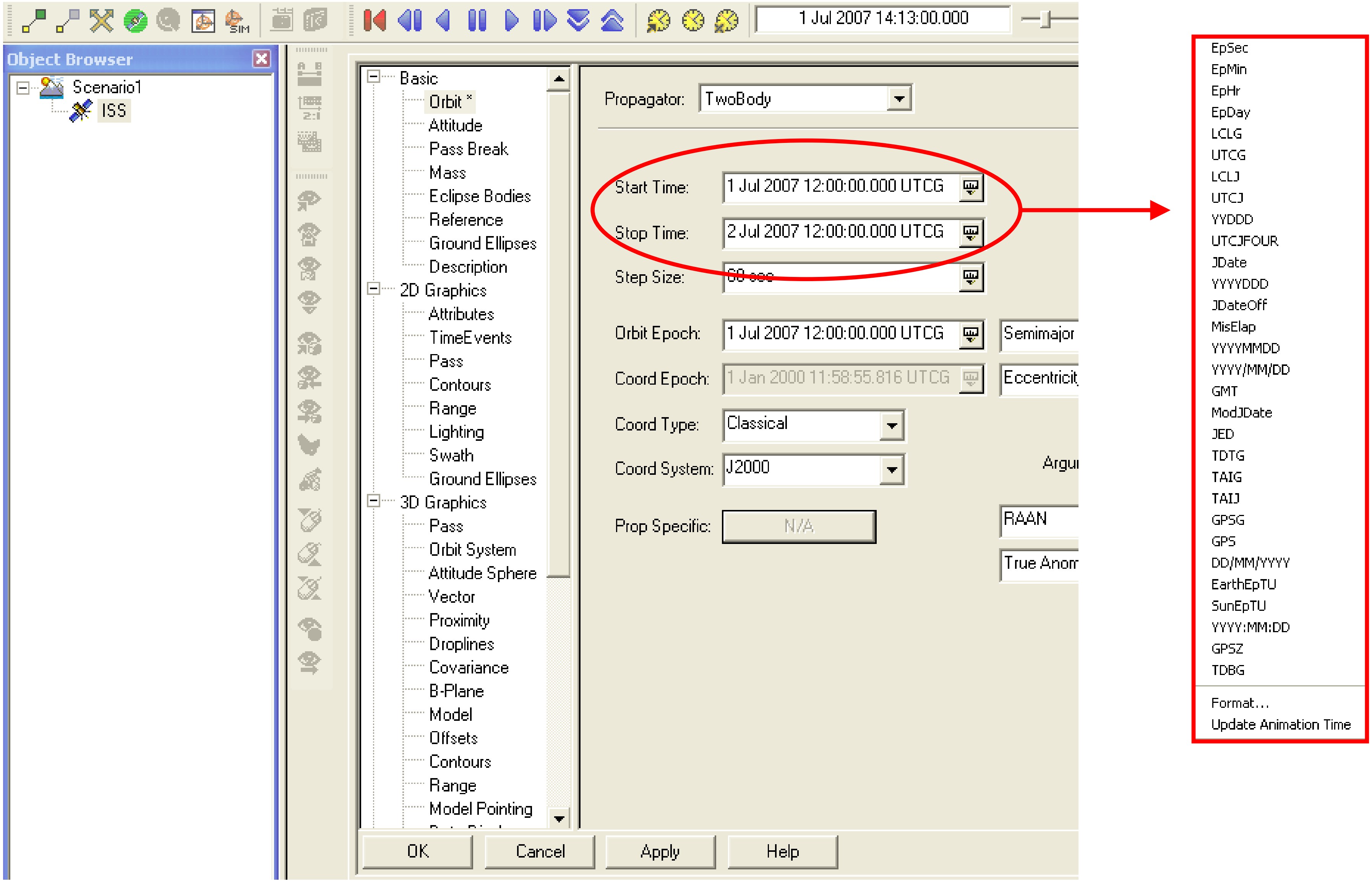
an error of 1 nanosecond translates into an error of 30cm in the distance.

What time is it? Well, no one knows for sure

guardian.co.uk

As the Earth spins slower, methods of telling time diverge. Experts warn this could end in disaster

Complexity: STK



Time in Astrodynamics (See Your Project)

Orbital parameters of the Sun :

$$a = 149\,600\,000 \text{ km} \quad e = 0.016709$$

$$i = 0.0000° \qquad \Omega + \omega = 282.9400°$$

$$M = 357.5256° + 35999.049° \times T$$

where

$$T = \frac{(JD - 2451545.0)}{36525.0} = \frac{JD2000}{365.25}$$

Ecliptic longitude of and distance to the Sun :

$$\lambda_{Sun} = \Omega + \omega + M + 6892'' \sin M + 72'' \sin 2M + 1.3972° \times T$$

$$r_{Sun} = (149.619 - 2.499 \cos M - 0.021 \cos 2M) \times 10^6 \text{ km}$$

Cartesian coordinates of the Sun :

$$\mathbf{r}_{Sun} \equiv \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{Sun} = \begin{pmatrix} r_{Sun} \cos \lambda_{Sun} \\ r_{Sun} \sin \lambda_{Sun} \cos \varepsilon \\ r_{Sun} \sin \lambda_{Sun} \sin \varepsilon \end{pmatrix}, \quad \varepsilon = 23.43929111°$$

The Julian Date

The Julian day number is the number of days since noon January 1, 4713 BC $\rightarrow$ Continuous time scale and no negative dates.

For historical reasons, the Julian day begins at noon, and not midnight, so that astronomers observing the heavens at night do not have to deal with a change of date.

The number of days between two events is found by subtracting the Julian day of one from that of the other.

Forward Computation of the Julian Date

Conversion from conventional time (YYYY, MM, DD, hh:mm:ss.ss) to Julian Date:

$$JD = 367.YYYY - \text{floor}\left\{\frac{7}{4}\left[YYYY + \text{floor}\left(\frac{MM+9}{12}\right)\right]\right\} +$$
$$\text{floor}\left(\frac{275.M}{9}\right) + DD + 1721013.5 + \frac{1}{24}\left[\frac{1}{60}\left(\frac{ss}{60} + mm\right) + hh\right]$$

Valid for the period 1st March 1900 and 28th February 2100

Backward Computation of the Julian Date

A.1.2 Calendar Date from the Modified Julian Date

The computation of the calendar date from the Modified Julian Date requires a number of intermediate steps. First, the integer Julian Day (i.e. the Julian Date at noon) is determined:

$$a = [\text{MJD}] + 2400001 \quad . \tag{A.7}$$

At the same time the fraction of day, q, is given by the decimal part of the Modified Julian Date:

$$q = \text{MJD} - [\text{MJD}] \quad . \tag{A.8}$$

Two auxiliary quantities b and c are defined as

$$b = \begin{cases} 0 & \text{if } a < 2299161 \text{ (Julian calendar)} \\ [(a-1867216.25)/36524.25] & \text{otherwise (Gregorian calendar)} \end{cases} \tag{A.9}$$

and

$$c = \begin{cases} a + 1524 & \text{if } a < 2299161 \text{ (Julian calendar)} \\ a + b - [b/4] + 1525 & \text{otherwise (Gregorian calendar)} \quad . \end{cases} \tag{A.10}$$

The next step is to calculate the auxiliary quantities

$$d = [(c-121.1)/365.25] \quad , \tag{A.11}$$

$$e = [365.25d] \tag{A.12}$$

and

$$f = [(c-e)/30.6001] \quad . \tag{A.13}$$

Finally, the calendar date is obtained from the following three steps: the day of the month (D) is given by

$$D = c - e - [30.6001\,f] + q \quad , \tag{A.14}$$

the month of the year (M) follows from

$$M = f - 1 - 12[f/14] \tag{A.15}$$

and the year (Y) in astronomical reckoning is determined by

$$Y = d - 4715 - [(7+M)/10] \quad . \tag{A.16}$$

It is again possible to simplify the computation somewhat if only a limited time interval is considered. E.g. the computation of the auxiliary quantities a, b, and c can be focussed into $c = [(\text{JD}+0.5)] + 1537$ if only the interval March 1900 until 2100 is taken into account.

$$MJD = JD - 2{,}400{,}000.5$$

Computation of Elapsed Time

Find the elapsed time between 4 October 1957 at 19:26:24 UTC and 12 May 2004 at 14:45:30 UTC

4 October 1957 at 19:26:24 UTC: 2436116.3100 days

12 May 2004 at 14:45:30 UTC: 2453138.11493056 days

→ The elapsed time is 17021.805 days

Standard Epoch Used Today: J2000

To lessen the magnitude of the Julian date, a constant offset can be introduced. A different reference epoch 1st January 2000 at noon is used:

$$J2000 = JD - 2451545$$

"Everyday" Time Systems

Are conventional local time systems adequate for orbital mechanics ?

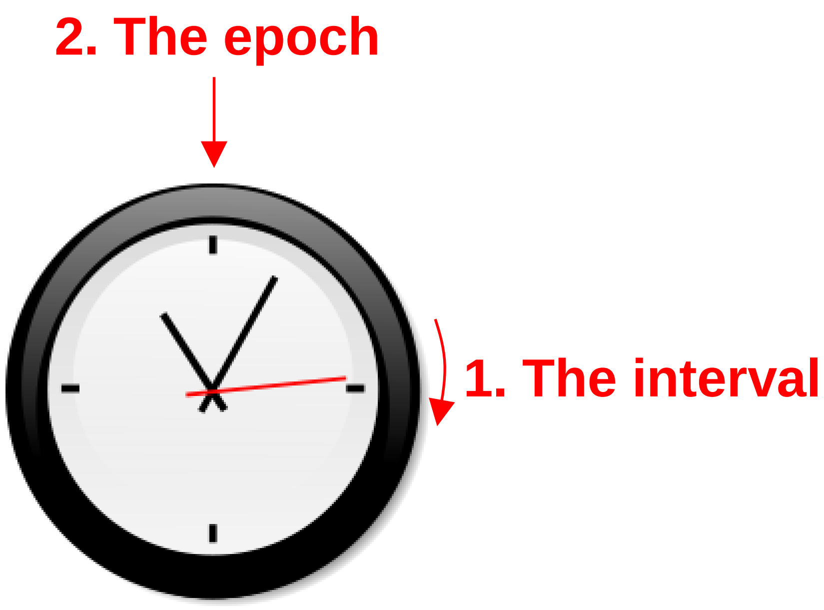
⇒ They depend on the user's position on Earth.

⇒ They are in a format (Y/M/D/H/M/S) that does not lend itself to use in a computer-implemented algorithm. For instance, what is the difference between any two dates ?

Objective of this section:

What would be a meaningful time system ?

What are the Ingredients of a Time System ?



1. The interval (a time reckoner): a repeatable phenomenon whose motion or change of state is observable and obeys a definite law.

2. The epoch (a time reference) from which to count intervals

Historical Perspective

From remote antiquity, the celestial bodies have been the fundamental reckoners of time (e.g. rising and setting of the Sun).

Sundials were among the first instruments used to measure the time of the day. The Egyptians divided the day and night into 12h each, which varied with the seasons (unequal seasonal hour)

Historical Perspective

It was not until the 14th century that an hour of uniform length became customary due to the invention of mechanical clocks.

Quartz-crystal clocks were developed in the 1920s.

The first atomic clock was constructed in 1948, and the first caesium atomic clock in 1955.

Quantum Clocks Soon ?

ULTRA-PRECISE QUANTUM-LOGIC CLOCK TRUMPS OLD ATOMIC CLOCK

Scientists have built a clock which is 37 times more precise than the existing international standard.

The quantum-logic clock, which detects the energy state of a single aluminum ion, keeps time to within a second every 3.7 billion years. The new timekeeper could one day improve GPS or detect the slowing of time predicted by Einstein's theory of general relativity.

https://www.wired.com/2010/02/quantum-logic-atomic-clock/

Tow Important Time Scales

1. *Universal time*: the time scale based on the rotation of the Earth on its axis.

2. *Atomic time*: the time scale based on the quantum mechanics of the atom.

Can We Use the Real Sun ?

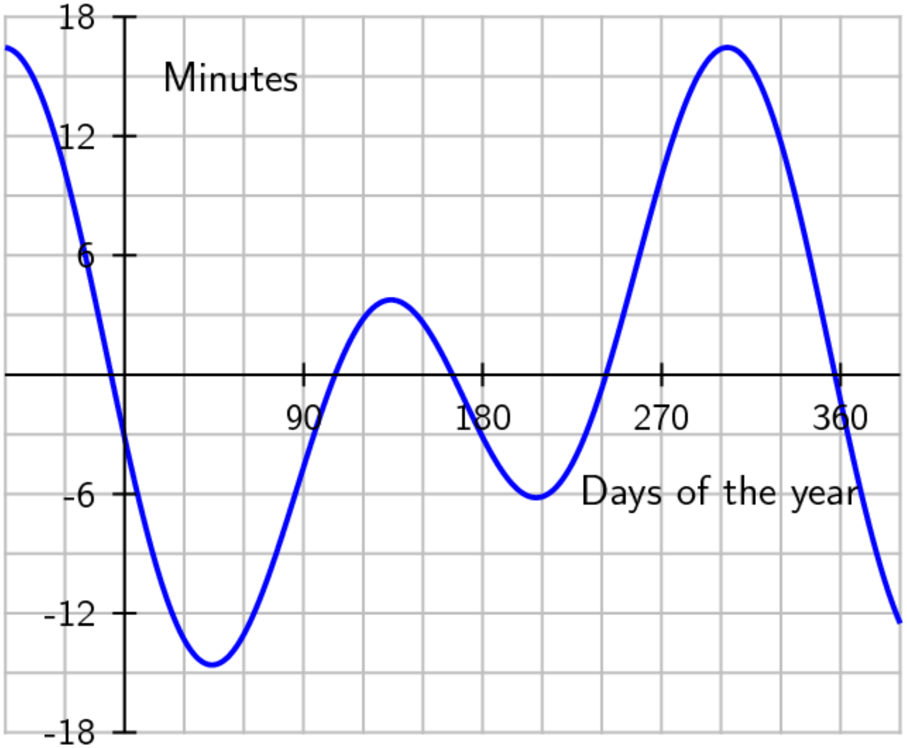
Apparent solar time, as read directly by a sundial, is the local time defined by the actual diurnal motion of the Sun.

Apparent solar day is the time required for the sun to lie on the same meridian.

Due to the eccentricity of Earth's orbit, the length of the apparent solar day varies throughout the year.

$\Rightarrow$ The real sun is not well suited for time reckoning purposes.

Apparent and Mean Solar Days



Approximation where E is in minutes, sin and cos in degrees, and N is the day number:

$$E = 9.87 \sin 2B - 7.53 \cos B - 1.5 \sin B$$

$$B = \frac{360^\circ (N - 81)}{365}$$

Equation of time

Can We Use a Fictitious Sun ?

At noon the fictitious sun lies on the Greenwich meridian.

A mean solar day comprises 24 hours. It is the time interval between successive transits of a fictitious mean sun over a given meridian. A constant velocity in the motion about the sun is therefore assumed.

The mean solar second can be defined as 1/86400 of a **mean solar day**.

Universal Time

Universal time is today’s realization of a **mean solar time** (introduced in 1920s).

It is the same everywhere on Earth.

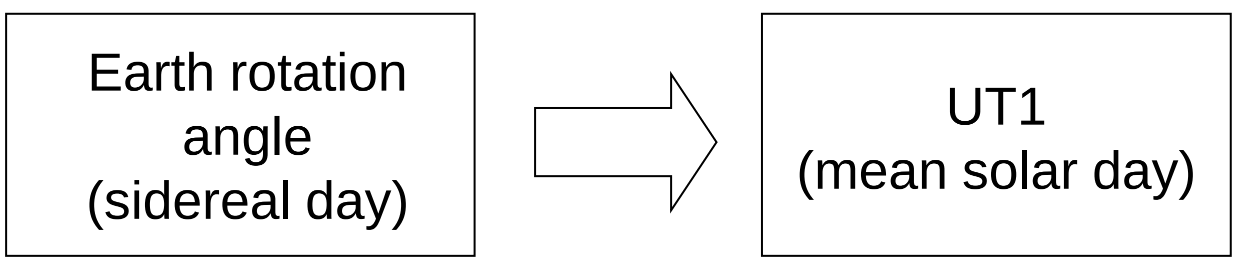
It is referred to the meridian of Greenwich and reckoned from midnight.

Universal Time UT1

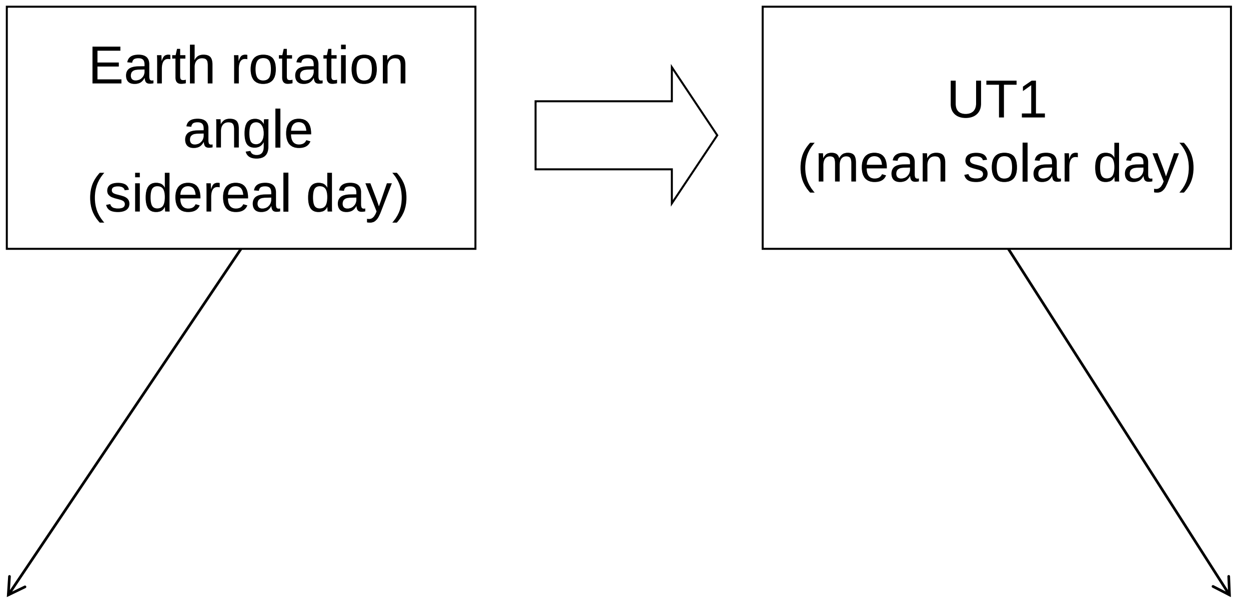
UT1 is the observed rotation of the Earth with respect to the mean sun.

It is based on the measurement of the Earth rotation angle with respect to an inertial reference frame (sidereal day).

A conversion from mean sidereal day to mean solar day is therefore necessary.



IAU2000 Definition of Universal Time UT1



$$ERA = 2\pi(0.7790572732640 + 1.00273781191135448\, T_u) \text{ radians}$$

IAU2000 Definition of Universal Time UT1

Earth rotation angle at J2000.0 UT1

Explanation 1

$$ERA = 2\pi(0.7790572732640 + 1.00273781191135448\, Tu) \text{ radians}$$

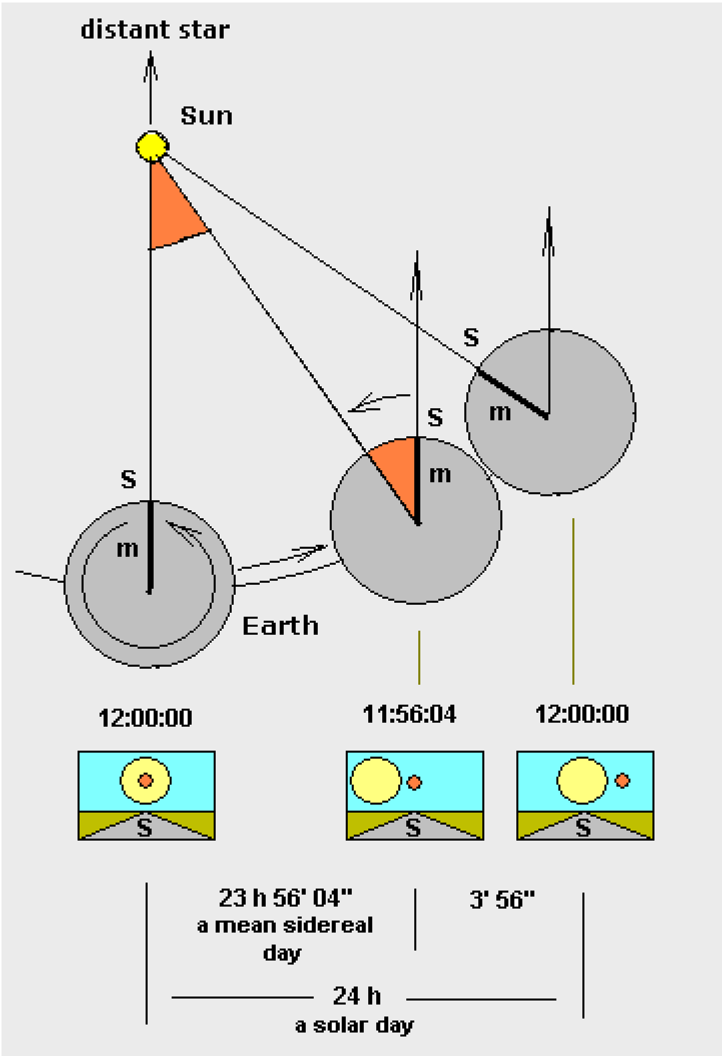
Earth rotation angle w.r.t. ICRF. Its time derivative is the Earth's angular velocity.

Explanation 3

Tu is directly related to UT1: Julian UT1 date - 2451545.0

Explanation 2

Explanation 1: Mean Solar Sidereal Days



1 solar day= 1.0027378119113544 8 sidereal day

Explanation 2: Julian Date

See previously.

Explanation 3: Accurate Determination of ERA

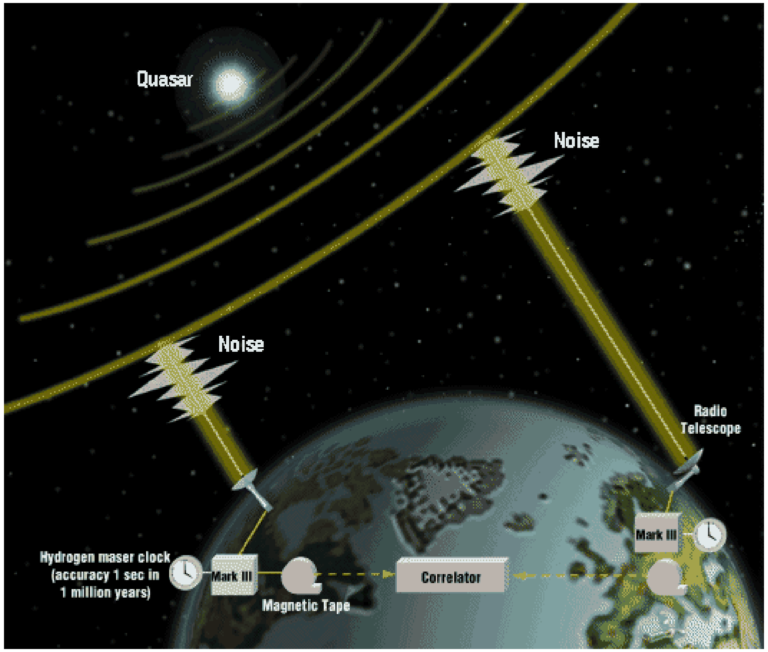
The most remote objects in the universe are quasars in a distance of about 3-15 billion light years. Because quasars are at such great distances that their motions across the sky are undetectable, they form a quasi-inertial reference frame, called the international celestial reference frame.

Quasars can be detected with very sensitive radiotelescopes.

By observing the diurnal motion of distant quasars (more precise than sun-based observations), it is possible to relate the position, orientation and rotation of the Earth to the inertial reference frame realized by these quasars.

Very Long Baseline Interferometry

A radio telescope with a cryogenic dual band S/X-band receiver (TIGO, Concepcion, Chili)



Can We Trust the Earth's Rotation ?

?

$$ERA = 2\pi(0.7790572732640 + 1.00273781191135448\ Tu)\ \text{radians}$$

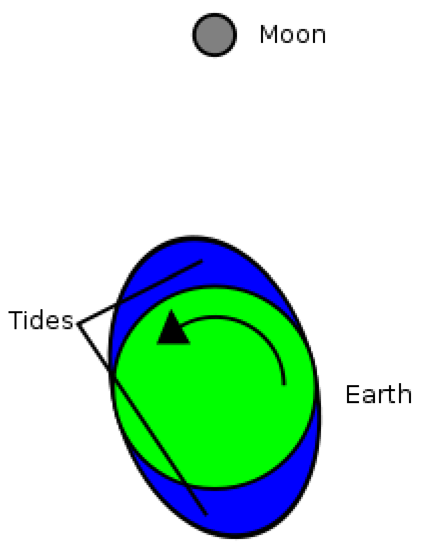
Can We Trust the Earth's Rotation ?

No !

- ⇒ The Earth's rotation rate is not uniform. It exhibits changes on the order of 2 milliseconds per day. Corals dating from 370 millions years ago indicate that the number of days was between 385 and 410.
- ⇒ There also exists random and seasonal variations.

In addition, the axis of rotation is not fixed in space.

Rotation Rate: Steady Deceleration (Cause 1)



Energy transfer from the Earth to the Moon

The Moon is at the origin of tides: the water of the oceans bulges out along both ends of an axis passing through the centers of the Earth and Moon.

The tidal bulge closely follows the Moon in its orbit, and the Earth rotates under this bulge in a day. Due to friction, the rotation drags the position of the tidal bulge ahead of the position directly under the Moon.

A substantial amount of mass in the bulge is offset from the line through the centers of the Earth and Moon. Because of this offset, there exists a torque which boosts the Moon in its orbit, and decelerates the rotation of the Earth.

Rotation Rate: Steady Deceleration (Cause 2)

In addition to this tidal acceleration of the Moon, the Earth is also slowing down due to tidal friction.

Tides stretch the oceans, and to a small extent, the solid mass of a planet or satellite. In one complete rotation, the planet material keeps deforming and relaxing. This takes energy away from the rotation, transforming it into heat.

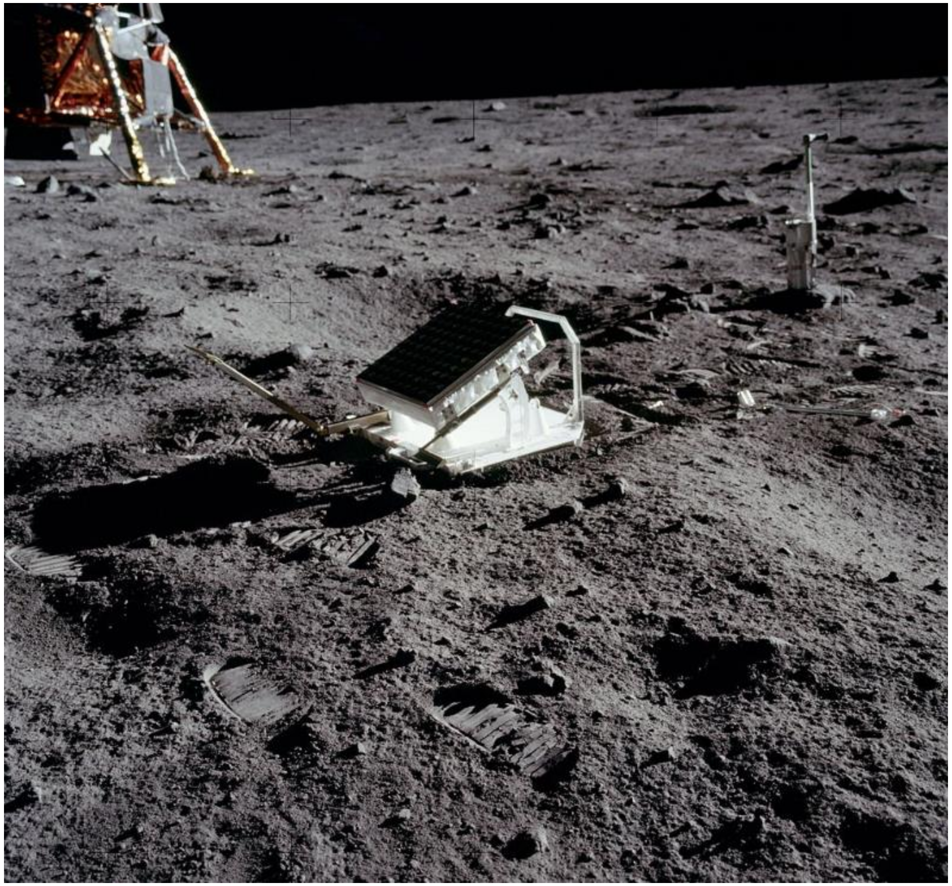
The Moon Is Moving Away from the Earth

The secular acceleration of the Moon is small but it has a cumulative effect on the Moon's position when extrapolated over many centuries.

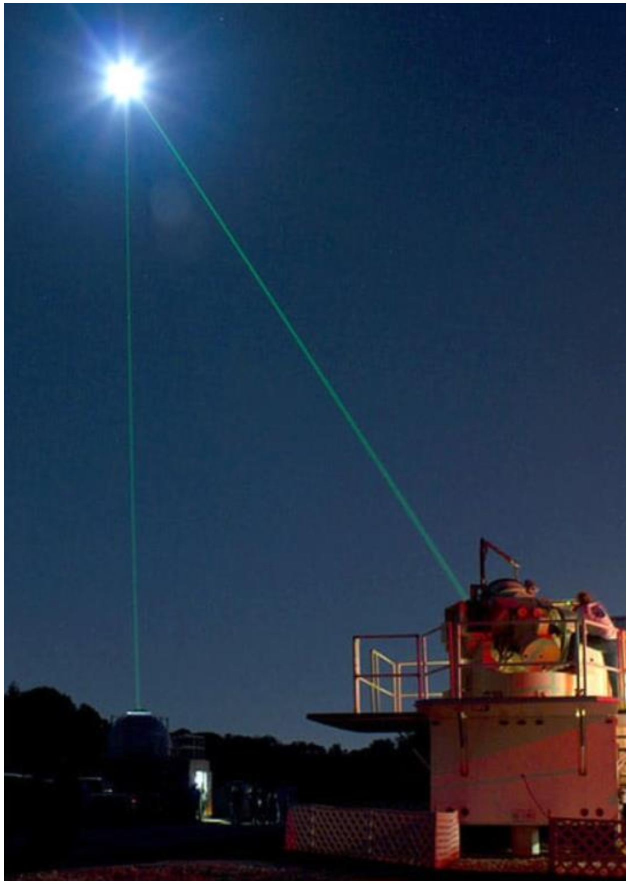
Direct measurements of the acceleration have been possible since 1969 using the Apollo retro-reflectors left on the Moon.

The results from Lunar Laser Ranging show that the Moon's mean distance from Earth is increasing by 3.8 cm per year.

Lunar Laser Ranging Experiment (Apollo 11)



Lunar Laser Ranging Experiment from the Apollo 11 mission



NASA Goddard
(Lunar Reconnaissance Orbiter)

Le séisme au Chili pourrait avoir raccourci les jours

Rédaction en ligne

mardi 02 mars 2010, 19:48

Le violent séisme qui a frappé le Chili a peut-être fait bouger l'axe de la Terre et, par conséquent, diminué la longueur du jour terrestre. La masse de la Terre étant désormais répartie autrement, notre planète tourne plus vite.

©AFP

Le séisme de magnitude 8,8 samedi au Chili pourrait avoir eu des effets sur l'axe de rotation de la Terre, entraînant un infime raccourcissement de la durée du jour, selon des scientifiques du Jet Propulsion Laboratory (JPL) de la NASA, l'agence spatiale américaine.

D'après des calculs préliminaires, issus d'une simulation informatique, le séisme du 27 février pourrait avoir entraîné un décalage de huit centimètres de l'axe de rotation terrestre, a expliqué mardi Richard Gross, du JPL, à Pasadena (Californie). Cela devrait provoquer un raccourcissement des jours de 1,26 microseconde, soit 1,26 millionième de seconde, ajoute-t-il. D'après Richard Gross, l'analyse des données du séisme permettra d'affiner les calculs.

Le phénomène n'est pas inédit. Comme dans tous les séismes majeurs, la Terre peut changer sa vitesse de rotation. Sous l'effet du séisme, la circonférence de la terre rétrécit très légèrement.

Le phénomène se retrouve lorsqu'une patineuse sur glace accélère sa rotation en fermant les bras et les rapprochant du corps. Très légère, cette accélération peut cependant être mesurée par satellite.

A titre d'exemple, le plus grand séisme du XXe siècle, d'une magnitude de 9,6 en 1960 au Chili, a fait diminuer la longueur du jour de huit microsecondes, selon une estimation des chercheurs.

Reste que l'atmosphère (frottements, jeu des masses d'air) et les marées océaniques ont une influence bien plus importante sur la durée du jour.

Quant à l'axe de la Terre, il varie naturellement tout le temps, décrivant en gros, à l'échelle d'une année, un cercle d'une dizaine de mètres. Le petit déplacement subi par l'axe de la Terre à cause du séisme chilien, estimé à huit centimètres, est donc moins élevé que le mouvement naturel de la Terre.

A ce mouvement mécanique s'ajoutent les mouvements des océans, des marées, l'influence de l'atmosphère, les éruptions volcaniques.

Pour provoquer un cataclysme et modifier réellement l'orbite terrestre, soulignent les sismologues, il faudrait une cause extérieure comme la collision avec un astéroïde.

What is Your Conclusion ?

We cannot “trust” the Earth’s rotation $\Rightarrow$ the length of one second of UT1 is not constant !

Its offset from atomic time is continually changing in a not completely predictable way.

International Atomic Time (TAI)

Since the advent of atomic time in 1955 there has been a steady transition from reliance on the Earth's rotation to the use of atomic time as the standard for the SI unit of duration (second).

The second is the duration of 9.192.631.770 cycles of the radiation corresponding to the transition between two hyperfine levels of the ground state of ^{133}Cs.

Weighted average of the time kept by about 300 atomic clocks in over 50 national laboratories worldwide.

Atomic Clocks: Stability and Accuracy

The caesium clock has high accuracy and good long-term stability.

The hydrogen maser has the best stability for periods of up to a few hours.

Is Atomic Time the Adequate Solution ?

No connection with the motion of the sun across the sky !

Physical and Astronomical Times

Astronomical clocks:

⇒ Related to everyday life.

⇒ Not consistent; the length of one second of UT is not constant. Typical accuracies ~10^{-8}.

Atomic clocks:

⇒ Consistent. Typical accuracies ~10^{-14}.

⇒ Not related to everyday life. If no adjustment is made, then within a millennium, local noon (i.e., the local time associated with the Sun’s zenith position) would occur at 13h00 and not 12h00.

Coordinated Universal Time (UTC)

The good practical compromise between atomic and universal times: it is the international standard on which civil time is based.

Its **time interval** corresponds to atomic time TAI:

$\Rightarrow$ It is accurate.

Its **epoch** differs by no more than 0.9 sec from UT1:

$\Rightarrow$ The mean sun is overhead on the Greenwich meridian at noon.

Leap Seconds

Leap seconds were introduced in 1971 to reconcile astronomical time, which is based on the rotation of the Earth, and physical time, which can be measured with great accuracy using atomic clocks.

Leap seconds are introduced to account for the fact that the Earth currently runs slow at 2 milliseconds per day and they ensure that the Sun continues to be overhead on the Greenwhich meridian at noon to within 1s.

Le Soir Article

Le Nouvel An en retard d'une seconde cette année

Rédaction en ligne

mercredi 31 décembre 2008, 09:19

Une seconde sera ajoutée à la dernière heure de 2008 pour refléter le ralentissement de la rotation de la Terre, sur fond de débat entre partisans de deux systèmes de mesure : le Greenwich Mean Time (« heure moyenne de Greenwich », GMT), une institution britannique, et le Temps atomique international (TAI), calculé près de Paris.

Le 31 décembre, à 23 heures, 59 minutes et 59 secondes en temps universel coordonné (UTC), une seconde supplémentaire sera ajoutée.

Not So Simple…

Record de vitesse de rotation de la Terre : "On pourrait être amené à retirer une seconde. Cela n'est jamais arrivé"

Le mercredi 29 juin 2022, la Terre a battu son record de vitesse de rotation.

La Rédaction

@

Publié le 05-08-2022 à 11h20 - Mis à jour le 05-08-2022 à 15h11

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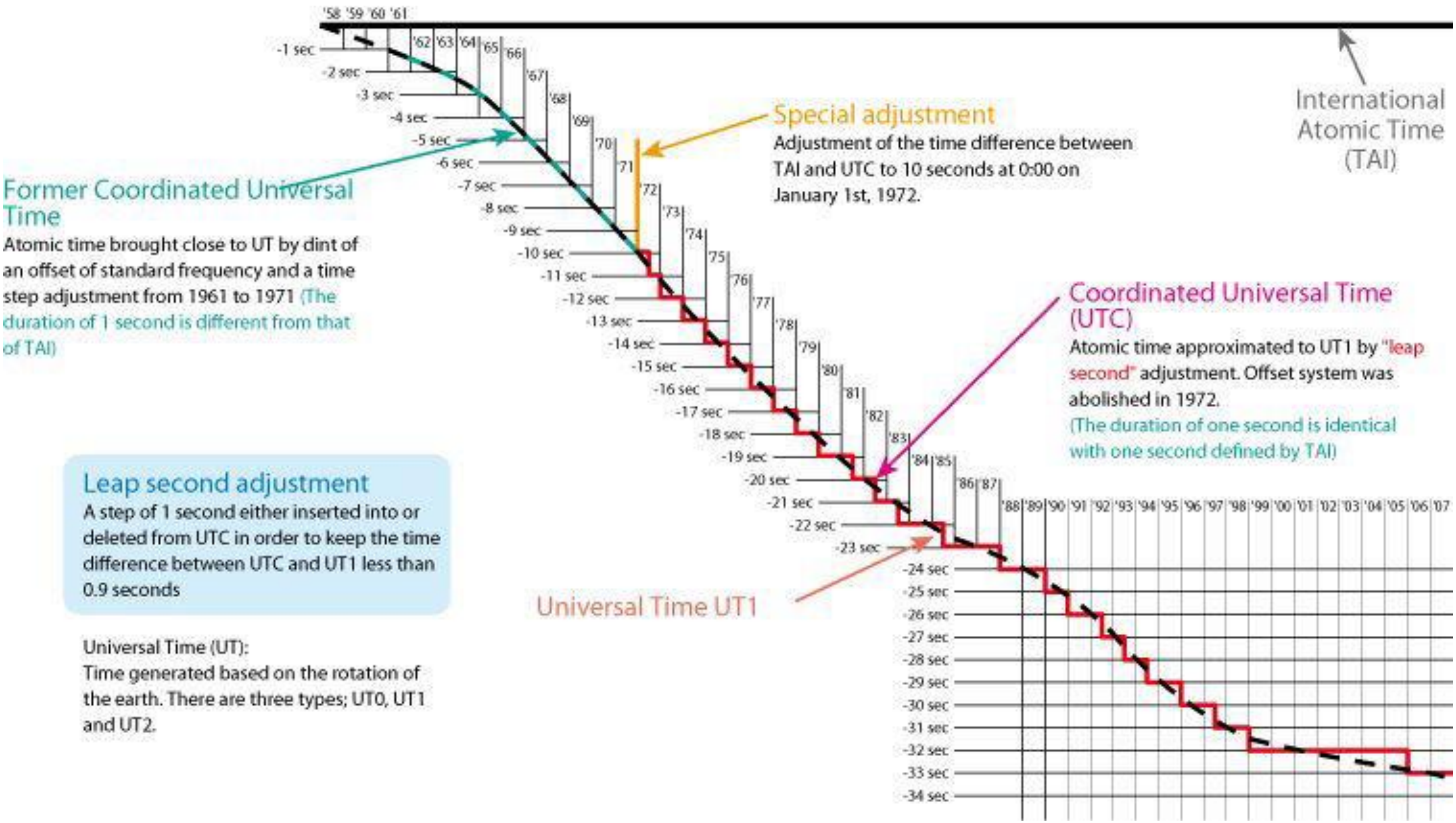
rotation de la Terre au journal Ouest-France, "*depuis 2016, on s'aperçoit que la vitesse de rotation de la Terre s'accélère, et donc la durée du jour diminue*". Avant cela, "*depuis 1930, raconte l'astronome,on observait une baisse de la vitesse de rotation de la Terreet donc une augmentation de la durée du jour. Cette tendance s'est inversée depuis sept ans sans qu'on puisse l'expliquer*".



'58 '59 '60 '61 '62 '63 '64 '65 '66 '67 '68 '69 '70 '71 '72 '73 '74 '75 '76 '77 '78 '79 '80 '81 '82 '83 '84 '85 '86 '87 '88 '89 '90 '91 '92 '93 '94 '95 '96 '97 '98 '99 '00 '01 '02 '03 '04 '05 '06 '07
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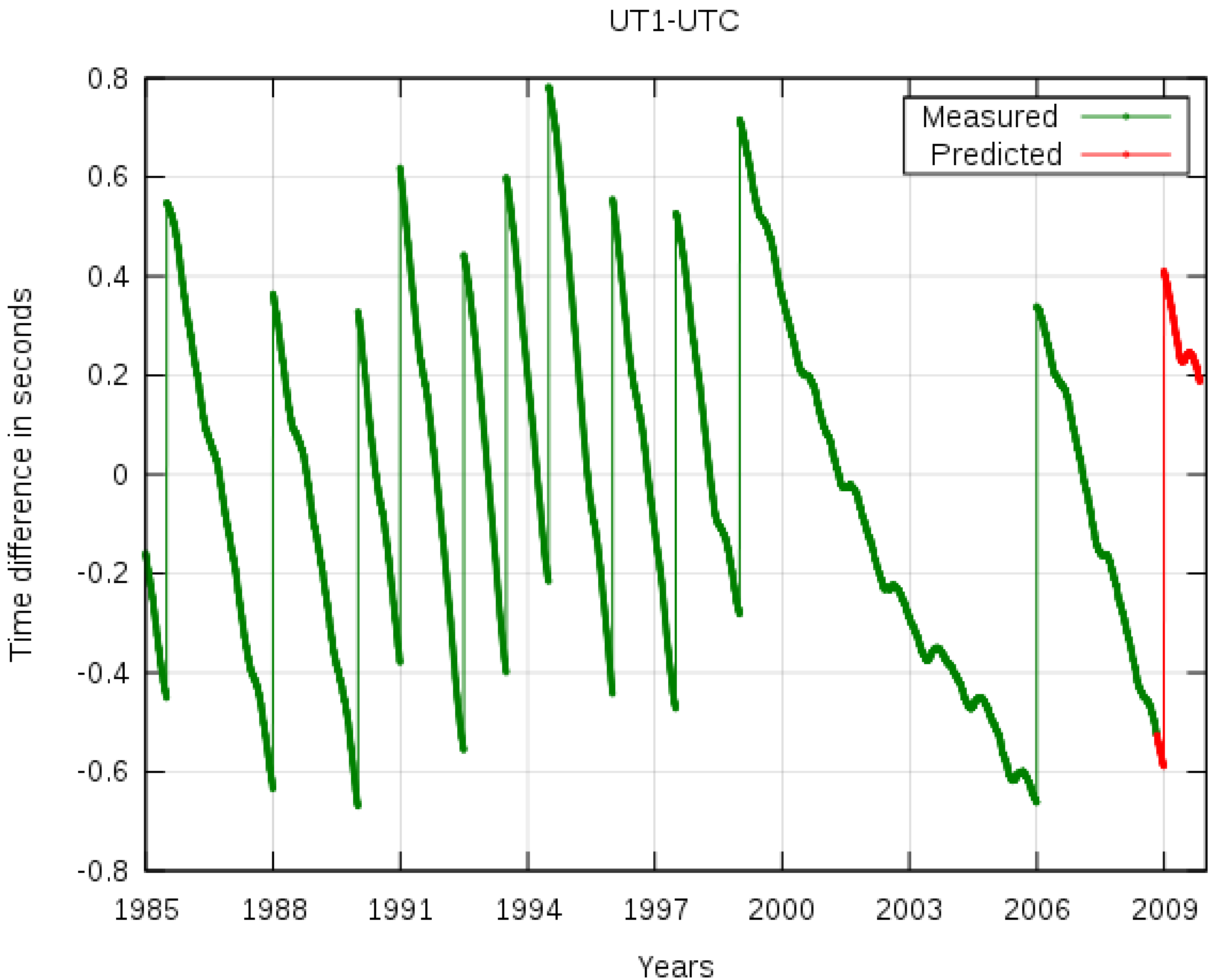
-32 sec
-33 sec
-34 sec
International Atomic Time (TAI)
Special adjustment
Adjustment of the time difference between TAI and UTC to 10 seconds at 0:00 on January 1st, 1972.
Former Coordinated Universal Time
Atomic time brought close to UT by dint of an offset of standard frequency and a time step adjustment from 1961 to 1971 (The duration of 1 second is different from that of TAI)
Coordinated Universal Time (UTC)
Atomic time approximated to UT1 by "leap second" adjustment. Offset system was abolished in 1972.
(The duration of one second is identical with one second defined by TAI)
Leap second adjustment
A step of 1 second either inserted into or deleted from UTC in order to keep the time difference between UTC and UT1 less than 0.9 seconds
Universal Time UT1
Universal Time (UT):
Time generated based on the rotation of the earth. There are three types; UT0, UT1 and UT2.

|DUT1 | =|UT1-UTC | <0.9s



Leap Seconds: Pros and Cons

This is currently the subject of intense debate (UT vs. TAI; i.e., UK vs. France).

+ Abandoning leap seconds would break sundials. In thousands of years, 16h00 would occur at 03h00. The British who have to wake up early in the morning to have tea…

  Astronomers

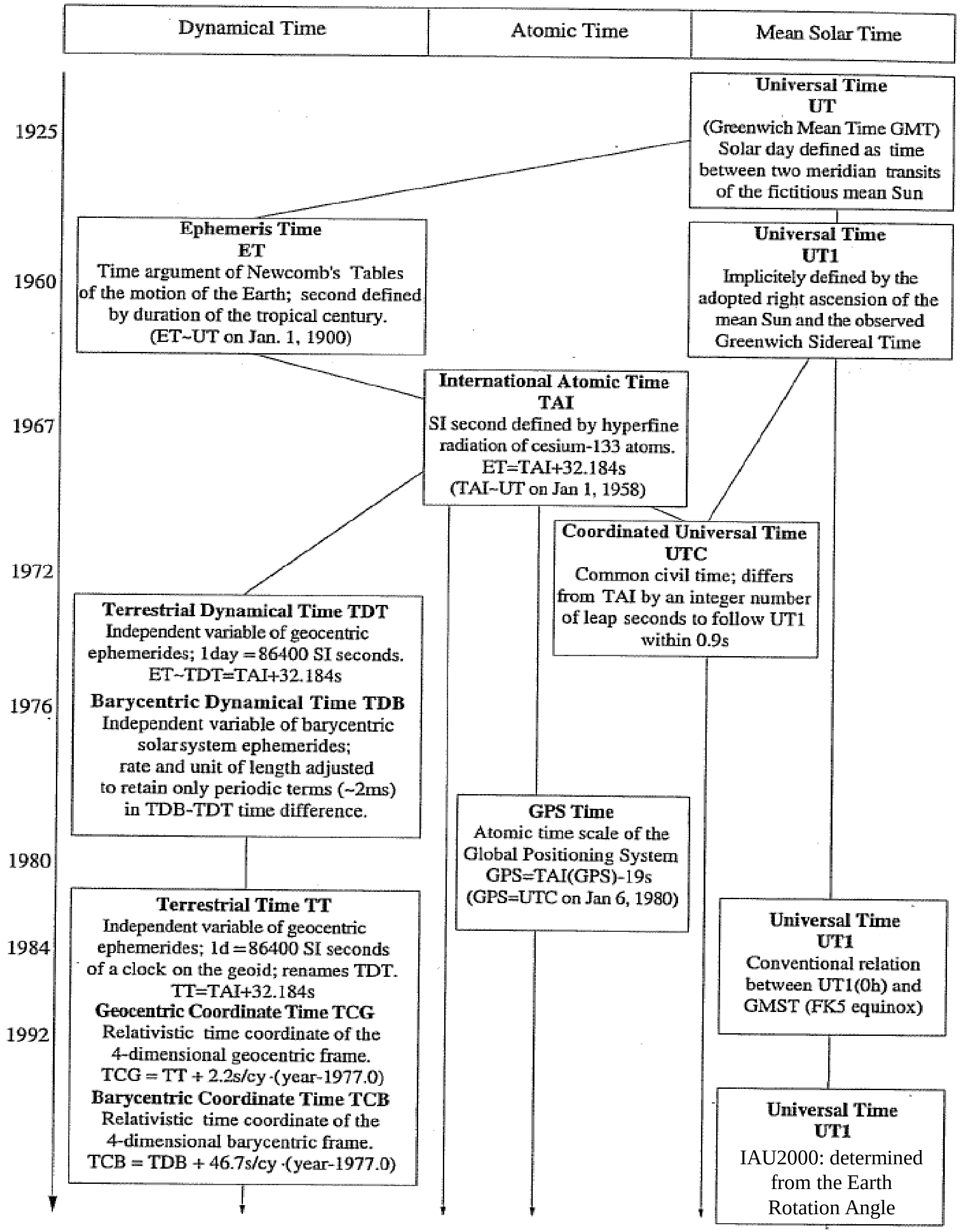
- Leap seconds are a worry with safety-critical real-time systems (e.g., air-traffic control — GPS' internal atomic clocks race ahead of UTC).

Yet More Time Systems !

GPS time: running ahead of UTC but behind TAI (it was set in 1980 based on UTC, but leap seconds were ignored since then).

Time standards for planetary motion calculations:

- ⇒ Terrestrial dynamic time: tied to TAI but with an offset of 32.184s to provide continuity with ephemeris time.
- ⇒ Barycentric dynamic time: similar to TDT but includes relativistic corrections that move the origin of the solar system barycenter.

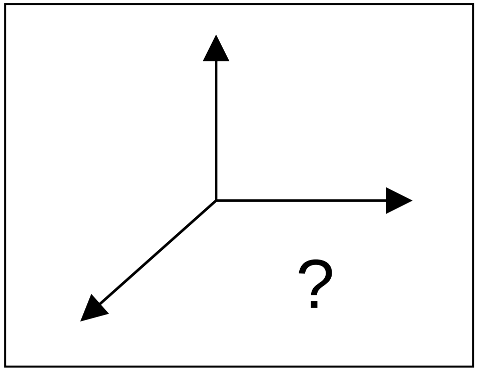

Dynamical Time
Atomic Time
Mean Solar Time
1925
1960
1967
1972
1976
1980
1984
1992
Universal Time
UT
(Greenwich Mean Time GMT)
Solar day defined as time between two meridian transits of the fictitious mean Sun
Ephemeris Time
ET
Time argument of Newcomb's Tables of the motion of the Earth; second defined by duration of the tropical century.
(ET~UT on Jan. 1, 1900)
Universal Time
UT1
Implicitely defined by the adopted right ascension of the mean Sun and the observed Greenwich Sidereal Time
International Atomic Time
TAI
SI second defined by hyperfine radiation of cesium-133 atoms.
ET=TAI+32.184s
(TAI~UT on Jan 1, 1958)
Coordinated Universal Time
UTC
Common civil time; differs from TAI by an integer number of leap seconds to follow UT1 within 0.9s
Terrestrial Dynamical Time TDT
Independent variable of geocentric ephemerides; 1day = 86400 SI seconds.
ET~TDT=TAI+32.184s
Barycentric Dynamical Time TDB
Independent variable of barycentric solarsystem ephemerides; rate and unit of length adjusted to retain only periodic terms (~2ms) in TDB-TDT time difference.
GPS Time
Atomic time scale of the Global Positioning System
GPS=TAI(GPS)-19s
(GPS=UTC on Jan 6, 1980)
Terrestrial Time TT
Independent variable of geocentric ephemerides; 1d = 86400 SI seconds of a clock on the geoid; renames TDT.
TT=TAI+32.184s
Geocentric Coordinate Time TCG
Relativistic time coordinate of the 4-dimensional geocentric frame.
TCG = TT + 2.2s/cy ·(year-1977.0)
Barycentric Coordinate Time TCB
Relativistic time coordinate of the 4-dimensional barycentric frame.
TCB = TDB + 46.7s/cy ·(year-1977.0)
Universal Time
UT1
Conventional relation between UT1(0h) and GMST (FK5 equinox)
Universal Time
UT1
IAU2000: determined from the Earth Rotation Angle

Further Reading

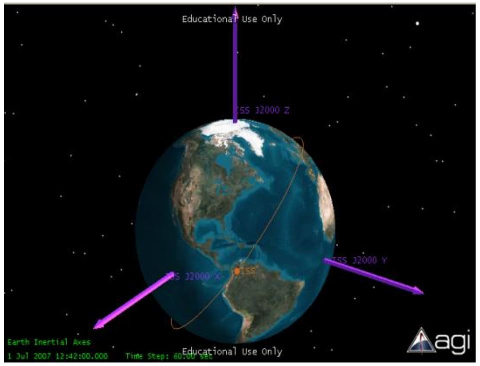
The leap second: its history and possible future

R. A. Nelson, D. D. McCarthy, S. Malys, J. Levine, B. Guinot, H. F. Fliegel, R. L. Beard and T. R. Bartholomew

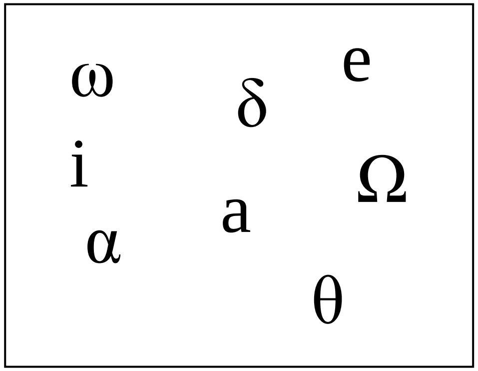
The Orbit in Space



Inertial frames

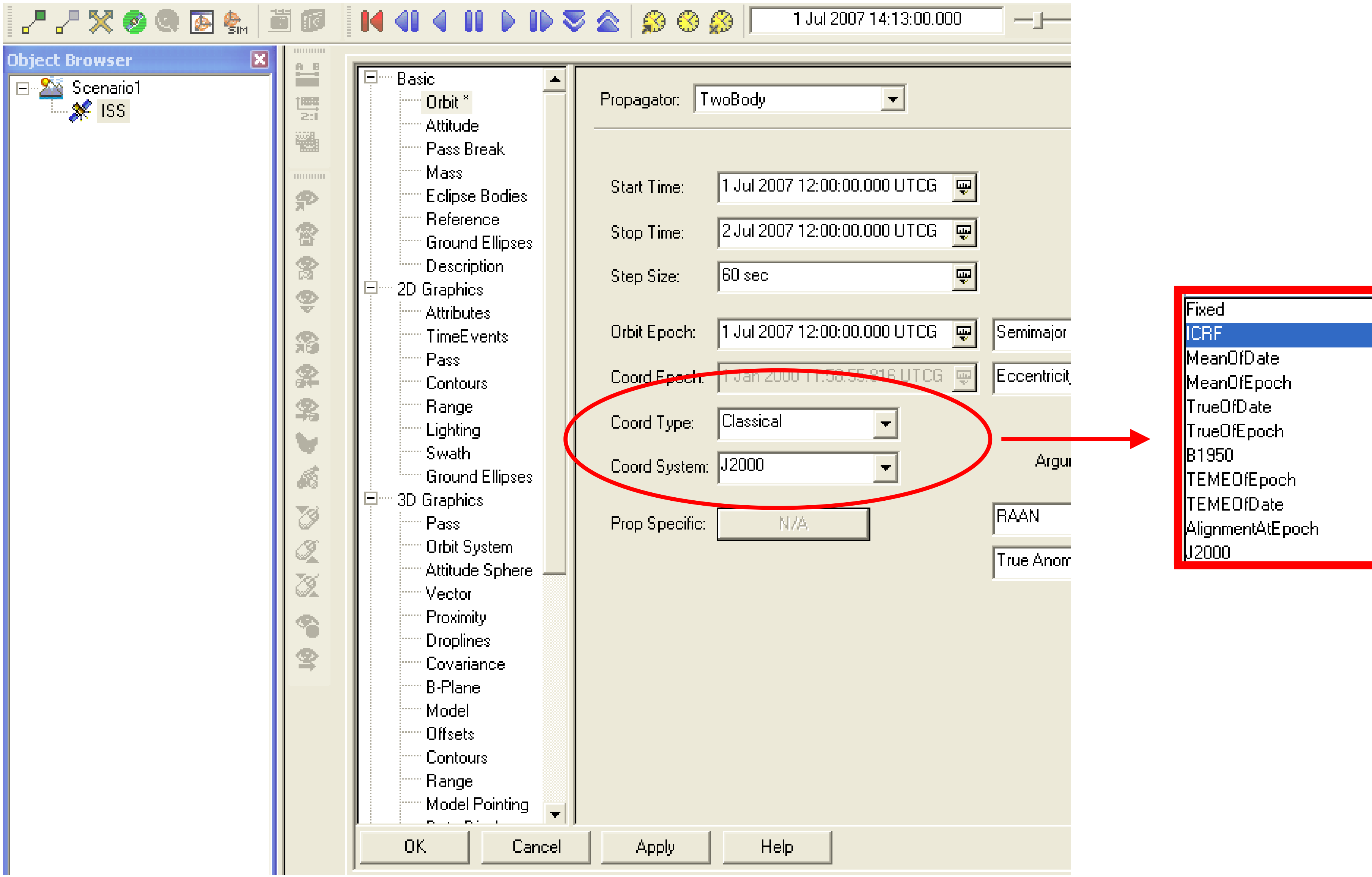


Coordinate systems



Coordinate types

Complexity of Coordinate Systems: STK



Importance of Inertial Frames

An inertial reference frame is defined as a system that is neither rotating nor accelerating relative to a certain reference point.

Suitable inertial frames are required for orbit description (remember that Newton’s second law is to be expressed in an inertial frame).

An inertial frame is also an appropriate coordinate system for expressing positions and motions of celestial objects.

Reference System and Reference Frame

Distinction between reference system and a reference frame:

1. A reference system is the complete specification of how a celestial coordinate system is to be formed. For instance, it defines the origin and fundamental planes (or axes) of the coordinate system.

2. A reference frame consists of a set of identifiable points on the sky along with their coordinates, which serves as the practical realization of a reference system.

International Celestial Reference System (ICRS)

The ICRS is the reference system of the International Astronomical Union (IAU) for high-precision astronomy.

Its origin is located at the barycenter of the solar system.

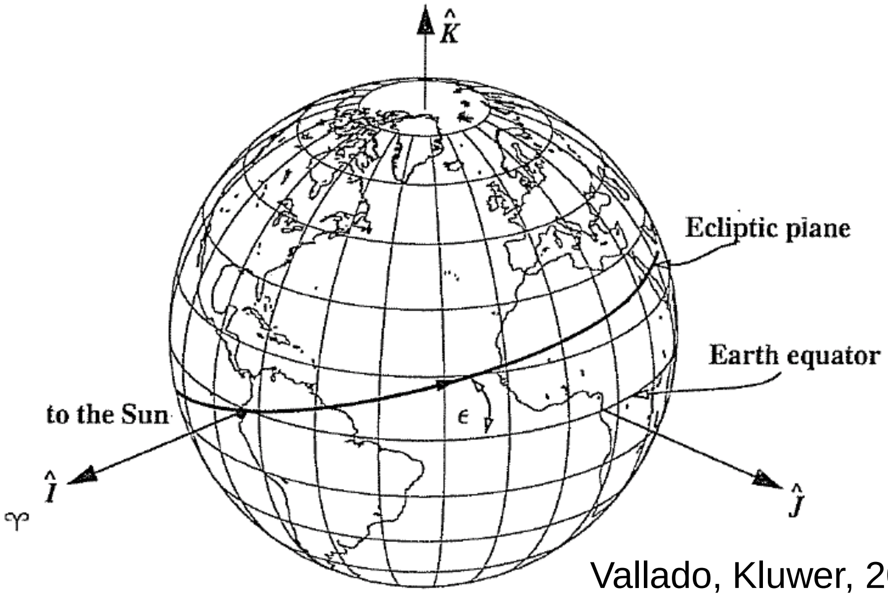
Definition of non-rotating axes:

1. The celestial pole is the Earth’s north pole (or the fundamental plane is the Earth's equatorial plane).

2. The reference direction is the vernal equinox (point at which the Sun crosses the equatorial plane moving from south to north).

3. Right-handed system.

Vernal Equinox ?



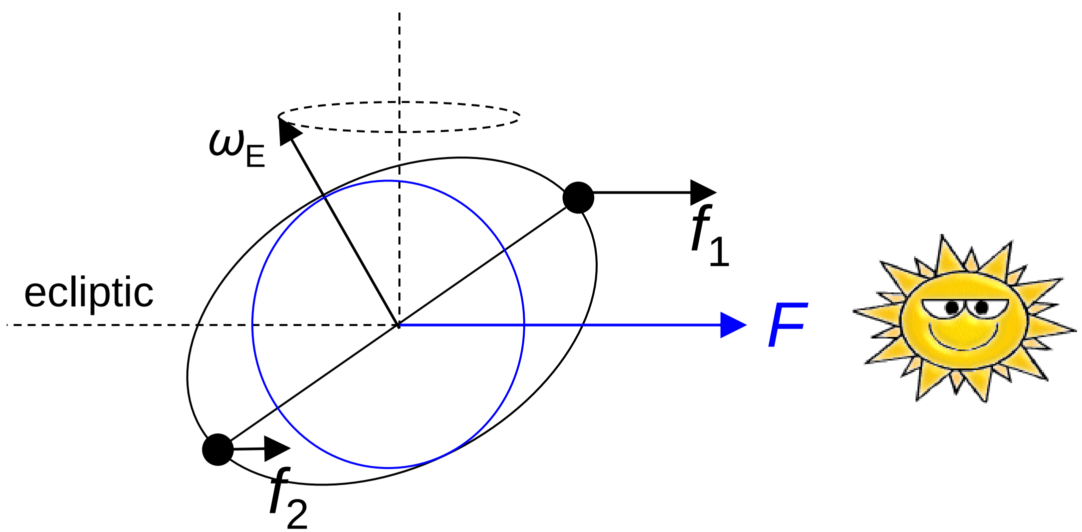
Vallado, Kluwer, 2001.

The vernal equinox is the intersection of the ecliptic and equator planes, where the sun passes from the southern to the northern hemisphere (First day of spring in the northern hemisphere).

Today, the vernal equinox points in the direction of the constellation Pisces, whereas it pointed in the direction of the constellation Ram during Christ’s lifetime. Why ?

Rotation Axis: Lunisolar Precession

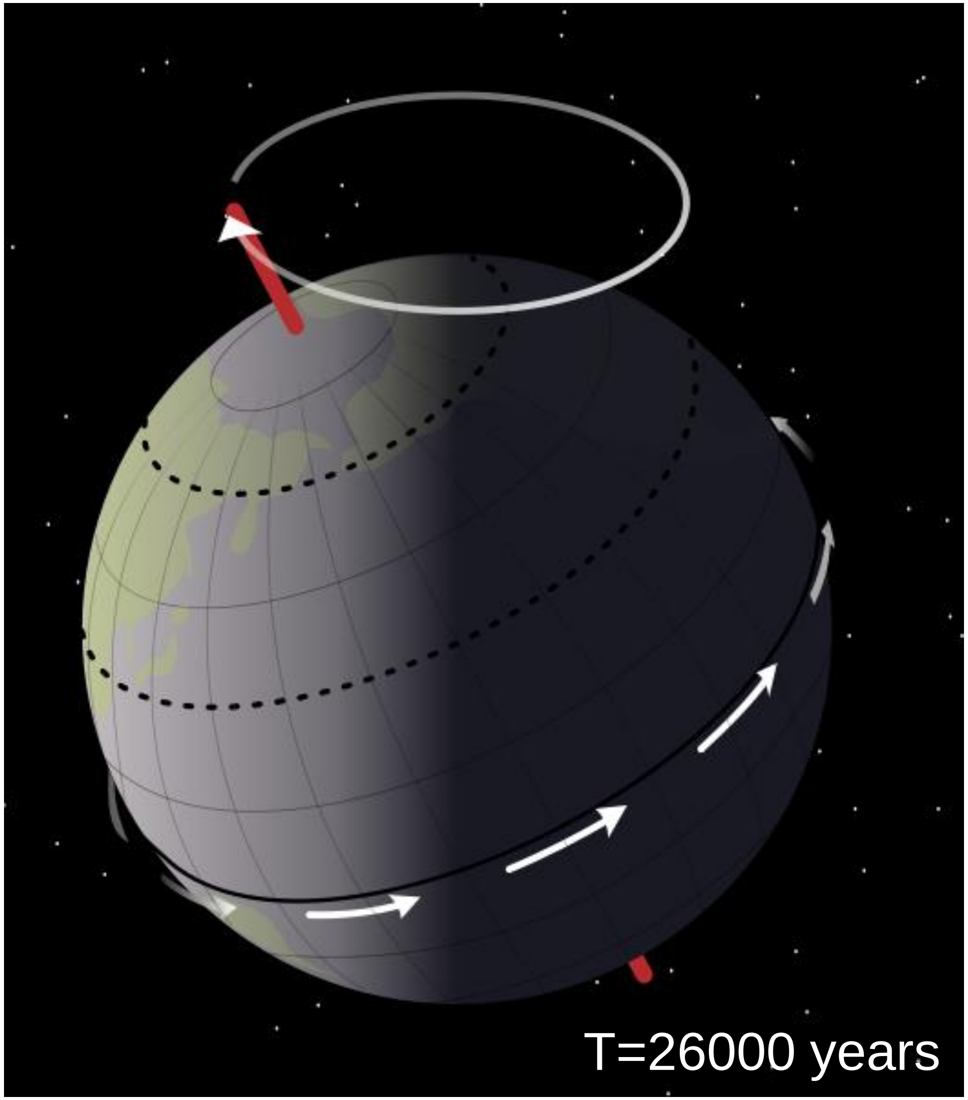
Because of the gravitational tidal forces of the Moon and Sun, the Earth's spin axis precesses westward around the normal to the ecliptic at a rate of 1.4°/century. The Earth's axis sweeps out a cone of 23.3 degrees in 26000 years.



F: dominant force on the spherical mass.

f_1, f_2: forces due to the bulging sides; $f_1 > f_2$, which implies a net clockwise moment.

Rotation Axis: Lunisolar Precession



Competition between two effects:

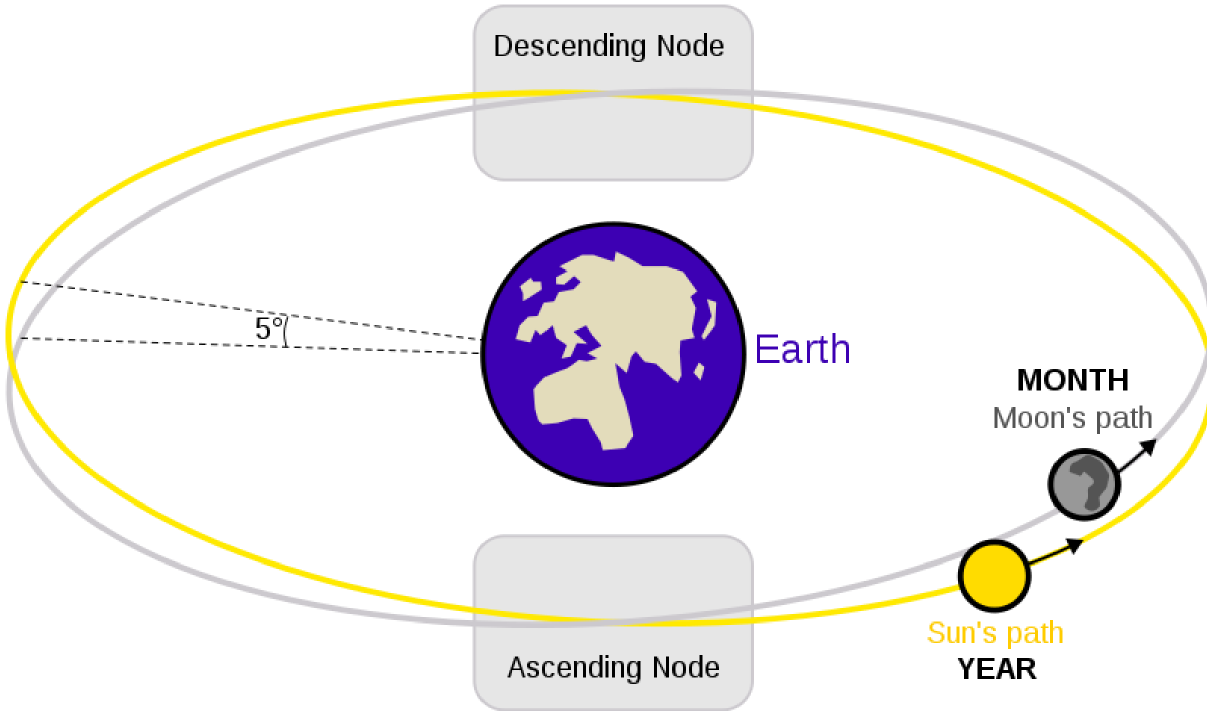
1. Gyroscopic stiffness of the spinning Earth (maintain orientation in inertial space).

2. Gravity gradient torque (pull the equatorial bulge into the plane of the ecliptic).

Rotation Axis: Nutation

The obliquity of the Earth varies with a maximum amplitude of 0.00025° over a period of 18.6 years.

This nutation is caused by the precession of the Moon's orbital nodes. They complete a revolution in 18.6 years.



Yet Another Disturbance: Polar Motion

Movement of Earth’s rotation axis across its surface.

Difference between the instantaneous rotational axis and the conventional international origin (CIO — a conventionally defined reference axis of the pole's average location over the year 1900).

The drift, about 20 m since 1900, is partly due to motions in the Earth's core and mantle, and partly to the redistribution of water mass as the Greenland ice sheet melts.

Yet Another Disturbance: Polar Motion

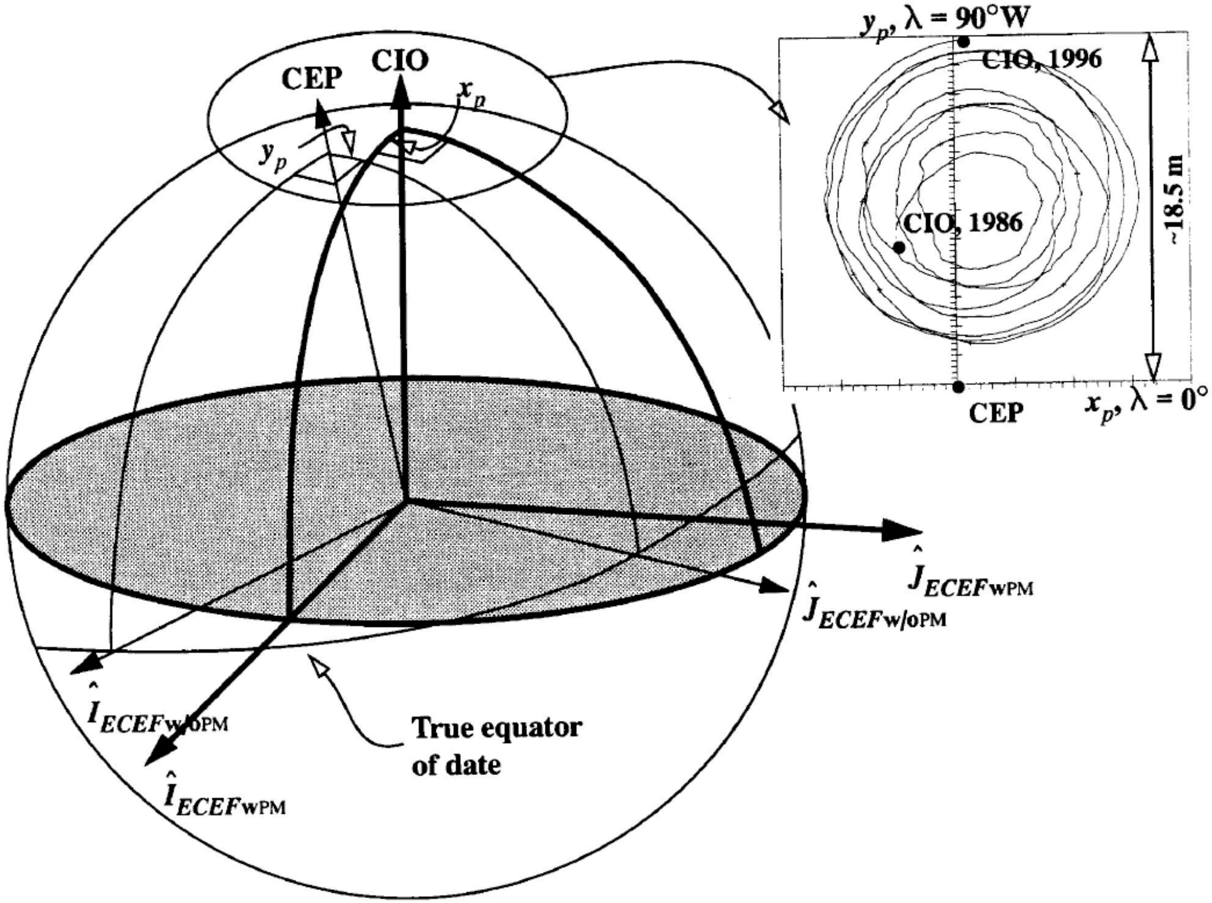
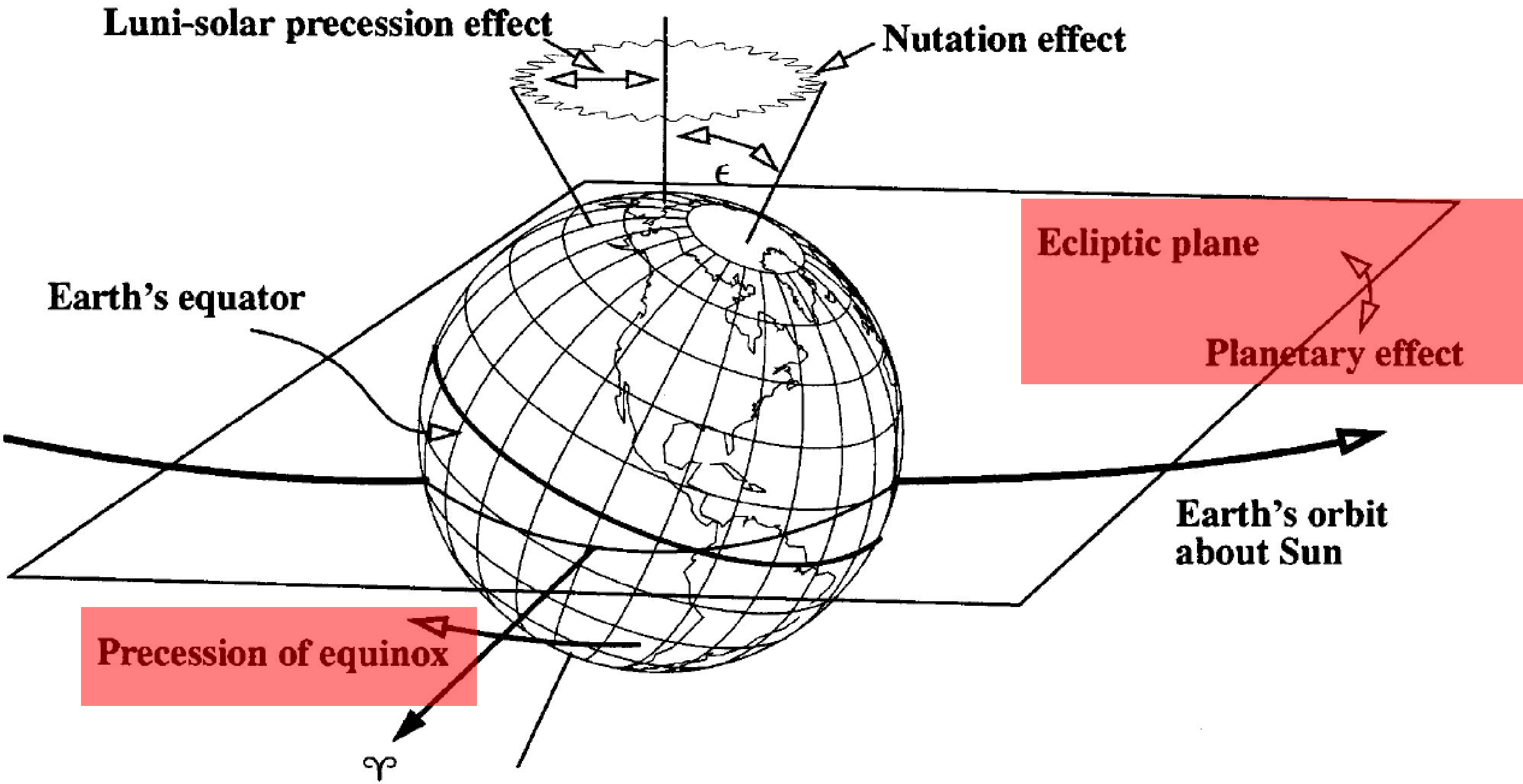


Figure 1-35. Transformation Geometry Due to Polar Motion. Accounting for polar motion takes into account the actual location of the Celestial Ephemeris Pole (CEP) over time. It moves from an *ECEF* system without polar motion through the CEP, to an *ECEF* system with polar motion using the Conventional International Origin (CIO). This correction changes the values very little, but highly accurate studies should include it. The inset plot shows the motion for the CIO from May 1986 to May 1996.

CIO: fixed with respect to the surface of the Earth

CEP: periodic motion (celestial ephemeris pole)

Complicated Motion of the Earth



Need To Specify a Date

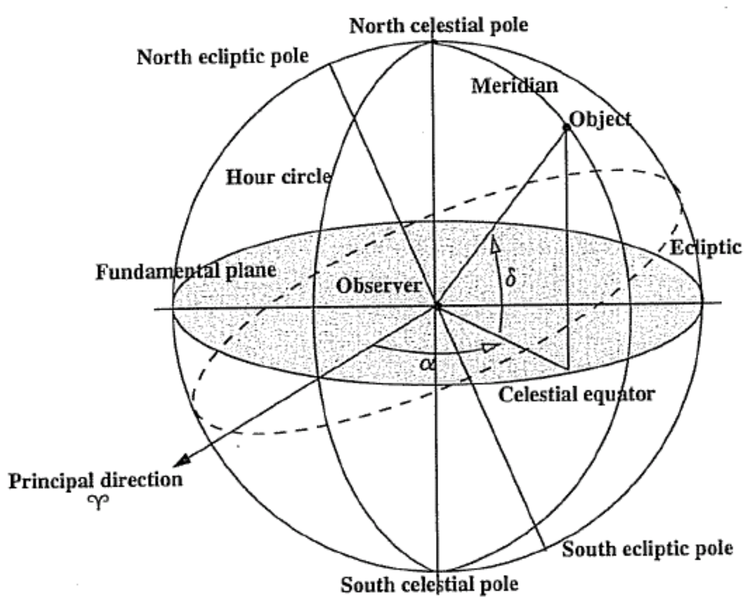
Because the ecliptic and equatorial planes are moving, the coordinate system must have a corresponding date:

"the pole/equator and equinox of [some date]".

For ICRS, the equator and equinox are considered at the epoch J2000.0 (January 1, 2000 at 11h58m56s UTC).

ICRS in Summary

Quasi-equatorial coordinates at the solar system barycenter !



An object is located in the ICRS using right ascension and declination

But how to realize ICRS practically ?

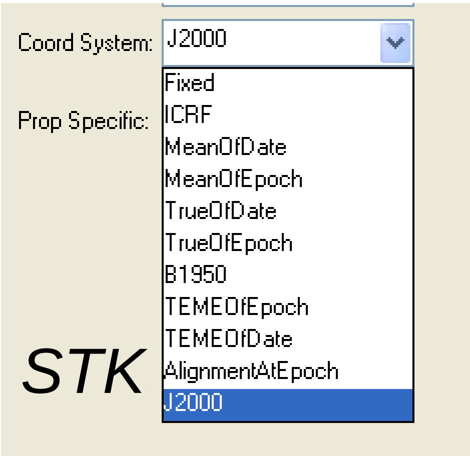
Previous Realizations: B1950 and J2000

B1950 and J2000 were considered the best realized inertial axes until the development of ICRF.

They exploit star catalogs (FK4 and FK5, respectively) which provide mean positions and proper motions for classical fundamental stars (optical measurements):

FK4 was published in 1963 and contained 1535 stars in various equinoxes from 1950 to 1975.

FK5 was an update of FK4 in 1988 with new positions for the 1535 stars.



1	0	8	23.265	+1.039	+29	5	25.58	-16.33	0	5	47.877	+1.036	+28	48	51.96	-16.33	43.31	0.7	2.0	33.00	1.3	3.1	2.06	A0p	+0.024	-11.7	358	BD+29	4	127
2	0	9	10.695	+6.827	+59	8	59.18	-18.09	0	6	29.765	+6.774	+58	52	26.54	-18.06	54.34	1.4	5.0	42.22	1.6	4.2	2.27	F5	+0.072	+11.8	432	BD+58	3	147
3	0	9	24.659	+1.186	-45	44	50.79	-18.11	0	6	52.788	+1.192	-46	1	23.36	-18.11	58.56	2.5	9.3	45.98	3.6	11.1	3.88	K0	+0.059	-9.2	496	CD-46	18	158
4	0	10	19.257	+0.074	+46	4	20.21	+0.03	0	7	42.779	+0.074	+45	47	38.71	+0.03	56.07	1.1	4.4	47.67	1.8	6.0	5.03	F0		-5.4	571	BD+45	17	169
5	0	11	34.437	+0.079	-27	47	59.12	+1.65	0	9	2.265	+0.079	-28	4	41.19	+1.65	56.47	1.6	6.5	46.63	2.9	9.7	5.42	K0		-5.7	720	CD-28	26	197
6	0	11	44.014	+1.412	-35	7	59.17	+11.86	0	9	11.739	+1.417	-35	24	46.32	+11.86	57.75	2.2	8.1	45.78	3.2	10.3	5.25	F5	+0.027	-1.7	739	CD-35	42	202
7	0	13	14.154	+0.019	+15	11	0.80	-1.20	0	10	39.483	+0.019	+14	54	20.50	-1.20	43.14	0.6	1.9	31.35	1.3	3.4	2.83	B2		+4.1	886	BD+14	14	238
9	0	19	25.674	-0.093	-08	49	26.14	-3.61	0	16	52.829	-0.093	-09	6	3.45	-3.61	43.93	0.8	2.5	32.95	1.6	4.8	3.56	K0	+0.010	+18.6	1522	BD-09	48	388
10	0	20	4.251	+26.778	-64	52	29.25	+116.39	0	17	28.799	+27.076	-65	10	6.35	+116.41	57.53	4.0	15.8	44.61	3.2	10.3	4.23	F8	+0.134	+8.7	1581	CP-65	13	401
11	0	25	45.056	+66.919	-77	15	15.40	+32.37	0	23	9.318	+68.429	-77	32	8.15	+32.37	43.89	6.4	17.2	28.53	2.6	6.8	2.80	G0	+0.153	+22.8	2151	CP-77	16	503
12	0	26	17.030	+1.833	-42	18	21.81	-39.57	0	23	49.051	+1.844	-42	34	38.31	-39.57	58.62	2.1	7.5	49.96	3.0	9.1	2.39	K0	+0.035	+74.6	2261	CD-42	116	519
13	0	30	2.362	+0.074	-03	57	26.39	-1.23	0	27	29.198	+0.074	-04	14	0.15	-1.23	42.48	0.7	2.4	30.56	1.5	4.5	5.72	K5		+4.7	2637	BD-04	54	584
14	0	30	22.661	-0.177	-23	47	15.72	+1.27	0	27	52.782	-0.177	-24	3	50.53	+1.27	53.36	1.4	5.1	46.67	2.6	9.5	5.19	A3	+0.012	+1.0	2696	CD-24	179	590
15	0	31	24.988	+1.449	-48	48	12.67	+1.75	0	29	0.619	+1.457	-49	4	47.11	+1.76	61.25	2.6	12.0	50.27	3.2	11.6	4.77	A2	+0.019	-5.0	2834	CD-49	115	619
16	0	32	59.982	+0.044	+62	55	54.40	-0.33	0	30	8.387	+0.044	+62	39	21.80	-0.33	51.76	1.7	6.0	41.93	1.7	4.9	4.16	B0		-2.3	2905	BD+62	102	645
17	0	36	58.291	+0.219	+53	53	48.92	-0.91	0	34	10.364	+0.218	+53	37	19.16	-0.91	55.12	1.2	4.6	42.60	1.7	5.0	3.66	B3		+2.1	3360	BD+53	105	727
18	0	36	52.858	+0.124	+33	43	9.63	-0.40	0	34	12.218	+0.124	+33	26	39.60	-0.40	53.90	0.9	3.4	45.10	1.8	5.6	4.36	B3		+8.7	3369	BD+32	101	729
19	0	38	33.350	-1.739	+29	18	42.30	-25.41	0	35	54.458	-1.732	+29	2	25.94	-25.42	52.82	0.8	3.1	45.56	1.5	5.1	4.37	G5	+0.031	-83.6	3546	BD+28	103	759
20	0	39	19.697	+1.060	+30	51	39.43	-9.15	0	36	38.890	+1.058	+30	35	15.48	-9.14	49.10	1.0	3.2	39.99	1.8	5.4	3.27	K2	+0.024	-7.3	3627	BD+30	91	774
21	0	40	30.450	+0.636	+56	32	14.46	-3.19	0	37	39.341	+0.632	+56	15	48.33	-3.18	50.70	1.2	3.8	32.85	1.5	3.4	2.23	K0	+0.009	-3.8	3712	BD+55	139	792
22	0	43	35.372	+1.637	-17	59	11.82	+3.25	0	41	4.844	+1.639	-18	15	38.66	+3.27	36.70	0.8	2.4	25.46	1.6	4.5	2.04	K0	+0.057	+13.1	4128	BD-18	115	965

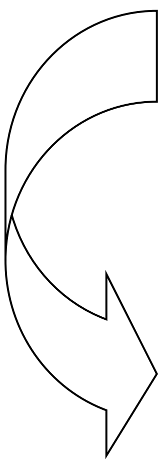
Fifth Fundamental Catalog (FK5), available on the web site

Byte-by-byte description of the file: catalog

Bytes	Format	Units	Labels	Explanations
1- 4	I4	---	FK5	*[1/1670]+ FK5 number
6- 7	I2	h	RAh	Right ascension, hours, Equinox=J2000, Epoch=J2000
9- 10	I2	min	RAm	Right ascension minutes (J2000.0)
12- 17	F6.3	s	RAs	*Right ascension seconds (J2000.0)
19- 25	F7.3	s/ha	pmRA	Proper motion in RA (J2000.0)
27	A1	---	DE-	Sign of declination (Dec) (J2000.0)
28- 29	I2	deg	DEd	Declination degrees (J2000.0)
31- 32	I2	arcmin	DEm	Declination arcminutes (J2000.0)
34- 38	F5.2	arcsec	DEs	*Declination arcseconds (J2000.0)
40- 46	F7.2	arcsec/ha	pmDE	Proper motion in DE (J2000.0)
48- 49	I2	h	RA1950h	Right ascension, hours Equinox=B1950, Epoch=B1950
51- 52	I2	min	RA1950m	Right ascension minutes (B1950.0)
54- 59	F6.3	s	RA1950s	*Right ascension seconds (B1950.0)
61- 67	F7.3	s/ha	pmRA1950	Proper motion in RA (B1950.0)
69	A1	---	DE1950-	Sign of declination (B1950.0)
70- 71	I2	deg	DE1950d	Declination degrees (B1950.0)
73- 74	I2	arcmin	DE1950m	Declination arcminutes (B1950.0)
76- 80	F5.2	arcsec	DE1950s	*Declination arcseconds (B1950.0)
82- 88	F7.2	arcsec/ha	pmDE1950	Proper motion in DE (B1950.0)
90- 94	F5.2	a	EpRA-1900	*Mean Epoch of observed RA
96- 99	F4.1	ms	e_RAs	*Mean error in RA
101-105	F5.1	ms/ha	e_pmRA	Mean error in pmRA
107-111	F5.2	a	EpDE-1900	*Mean Epoch of observed DE
113-116	F4.1	carcsec	e_DEs	*Mean error in Declination
118-122	F5.1	carcsec/ha	e_pmDE	Mean error in pmDE
124-128	F5.2	mag	Vmag	*V magnitude
129	A1	---	n_Vmag	*[VvD] Magnitude flag
131-137	A7	---	SpType	*Spectral type(s)
139-144	F6.3	arcsec	plx	*?Parallax
147-152	F6.1	km/s	RV	*?Radial velocity
155-159	A5	---	AGK3R	AGK3R number (Catalog <I/72>)

Star Catalogs: Limitations and Improvement

1. The uncertainties in the star positions of the FK5 are about 30-40 milliarcseconds over most of the sky.
2. A stellar reference frame is time-dependent because stars exhibit detectable motions.



1. Uncertainties of radio source positions are now typically less than one milliarcsecond, and often a factor of ten better.
2. Radio sources are not expected to show measurable intrinsic motion.

1	0	8	23.265	+1.039	+29	5	25.58	-16.33	0	5	47.877	+1.036	+28	48	51.96	-16.33	43.31	0.7	2.0	33.00	1.3	3.1	2.06	A0p	+0.024	-11.7	358	BD+29	4	127
2	0	9	10.695	+6.827	+59	8	59.18	-18.09	0	6	29.765	+6.774	+58	52	26.54	-18.06	54.34	1.4	5.0	42.22	1.6	4.2	2.27	F5	+0.072	+11.8	432	BD+58	3	147
3	0	9	24.659	+1.186	-45	44	50.79	-18.11	0	6	52.788	+1.192	-46	1	23.36	-18.11	58.56	2.5	9.3	45.98	3.6	11.1	3.88	K0	+0.059	-9.2	496	CD-46	18	158
4	0	10	19.257	+0.074	+46	4	20.21	+0.03	0	7	42.779	+0.074	+45	47	38.71	+0.03	56.07	1.1	4.4	47.67	1.8	6.0	5.03	F0		-5.4	571	BD+45	17	169
5	0	11	34.437	+0.079	-27	47	59.12	+1.65	0	9	2.265	+0.079	-28	4	41.19	+1.65	56.47	1.6	6.5	46.63	2.9	9.7	5.42	K0		-5.7	720	CD-28	26	197
6	0	11	44.014	+1.412	-35	7	59.17	+11.86	0	9	11.739	+1.417	-35	24	46.32	+11.86	57.75	2.2	8.1	45.78	3.2	10.3	5.25	F5	+0.027	-1.7	739	CD-35	42	202
7	0	13	14.154	+0.019	+15	11	0.80	-1.20	0	10	39.483	+0.019	+14	54	20.50	-1.20	43.14	0.6	1.9	31.35	1.3	3.4	2.83	B2		+4.1	886	BD+14	14	238
9	0	19	25.674	-0.093	-08	49	26.14	-3.61	0	16	52.829	-0.093	-09	6	3.45	-3.61	43.93	0.8	2.5	32.95	1.6	4.8	3.56	K0	+0.010	+18.6	1522	BD-09	48	388
10	0	20	4.251	+26.778	-64	52	29.25	+116.39	0	17	28.799	+27.076	-65	10	6.35	+116.41	57.53	4.0	15.8	44.61	3.2	10.3	4.23	F8	+0.134	+8.7	1581	CP-65	13	401
11	0	25	45.056	+66.919	-77	15	15.40	+32.37	0	23	9.318	+68.429	-77	32	8.15	+32.37	43.89	6.4	17.2	28.53	2.6	6.8	2.80	G0	+0.153	+22.8	2151	CP-77	16	503
12	0	26	17.030	+1.833	-42	18	21.81	-39.57	0	23	49.051	+1.844	-42	34	38.31	-39.57	58.62	2.1	7.5	49.96	3.0	9.1	2.39	K0	+0.035	+74.6	2261	CD-42	116	519
13	0	30	2.362	+0.074	-03	57	26.39	-1.23	0	27	29.198	+0.074	-04	14	0.15	-1.23	42.48	0.7	2.4	30.56	1.5	4.5	5.72	K5		+4.7	2637	BD-04	54	584
14	0	30	22.661	-0.177	-23	47	15.72	+1.27	0	27	52.782	-0.177	-24	3	50.53	+1.27	53.36	1.4	5.1	46.67	2.6	8.5	5.19	A3	+0.012	+1.0	2696	CD-24	179	590
15	0	31	24.988	+1.449	-48	48	12.67	+1.75	0	29	0.619	+1.457	-49	4	47.11	+1.76	61.25	2.6	12.0	50.27	3.2	11.6	4.77	A2	+0.019	-5.0	2834	CD-49	115	619
16	0	32	59.982	+0.044	+62	55	54.40	-0.33	0	30	8.387	+0.044	+62	39	21.80	-0.33	51.76	1.7	6.0	41.93	1.7	4.9	4.16	B0		-2.3	2905	BD+62	102	645
17	0	36	58.291	+0.219	+53	53	48.92	-0.91	0	34	10.364	+0.218	+53	37	19.16	-0.91	55.12	1.2	4.6	42.60	1.7	5.0	3.66	B3		+2.1	3360	BD+53	105	727
18	0	36	52.858	+0.124	+33	43	9.63	-0.40	0	34	12.218	+0.124	+33	26	39.60	-0.40	53.90	0.9	3.4	45.10	1.8	5.6	4.36	B3		+8.7	3369	BD+32	101	729
19	0	38	33.350	-1.739	+29	18	42.30	-25.41	0	35	54.458	-1.732	+29	2	25.94	-25.42	52.82	0.8	3.1	45.56	1.5	5.1	4.37	G5	+0.031	-83.6	3546	BD+28	103	759
20	0	39	19.697	+1.060	+30	51	39.43	-9.15	0	36	38.890	+1.058	+30	35	15.48	-9.14	49.10	1.0	3.2	39.99	1.8	5.4	3.27	K2	+0.024	-7.3	3627	BD+30	91	774
21	0	40	30.450	+0.636	+56	32	14.46	-3.19	0	37	39.341	+0.632	+56	15	48.33	-3.18	50.70	1.2	3.8	32.85	1.5	3.4	2.23	K0	+0.009	-3.8	3712	BD+55	139	792
22	0	43	35.372	+1.637	-17	59	11.82	+3.25	0	41	4.844	+1.639	-18	15	38.66	+3.27	36.70	0.8	2.4	25.46	1.6	4.5	2.04	K0	+0.057	+13.1	4128	BD-19	115	965

Fifth Fundamental Catalog (FK5), available on the web site

Byte-by-byte description of the file: catalog

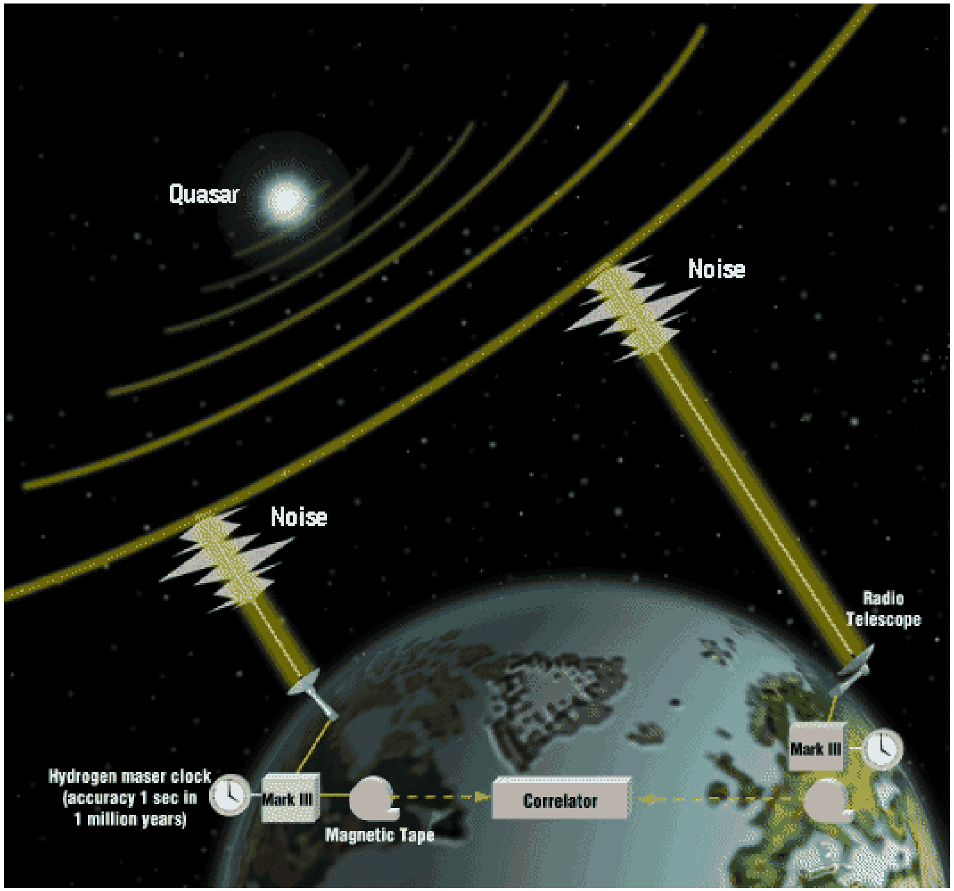
Bytes	Format	Units	Labels	Explanations
1- 4	I4	---	FK5	*[1/1670]+ FK5 number
6- 7	I2	h	RAh	Right ascension, hours, Equinox=J2000, Epoch=J2000
9- 10	I2	min	RAm	Right ascension minutes (J2000.0)
12- 17	F6.3	s	RAs	*Right ascension seconds (J2000.0)
19- 25	F7.3	s/ha	pmRA	Proper motion in RA (J2000.0)
27	A1	---	DE-	Sign of declination (Dec) (J2000.0)
28- 29	I2	deg	DEd	Declination degrees (J2000.0)
31- 32	I2	arcmin	DEm	Declination arcminutes (J2000.0)
34- 38	F5.2	arcsec	DEs	*Declination arcseconds (J2000.0)
40- 46	F7.2	arcsec/ha	pmDE	Proper motion in DE (J2000.0)
48- 49	I2	h	RA1950h	Right ascension, hours Equinox=B1950, Epoch=B1950
51- 52	I2	min	RA1950m	Right ascension minutes (B1950.0)
54- 59	F6.3	s	RA1950s	*Right ascension seconds (B1950.0)
61- 67	F7.3	s/ha	pmRA1950	Proper motion in RA (B1950.0)
69	A1	---	DE1950-	Sign of declination (B1950.0)
70- 71	I2	deg	DE1950d	Declination degrees (B1950.0)
73- 74	I2	arcmin	DE1950m	Declination arcminutes (B1950.0)
76- 80	F5.2	arcsec	DE1950s	*Declination arcseconds (B1950.0)
82- 88	F7.2	arcsec/ha	pmDE1950	Proper motion in DE (B1950.0)
90- 94	F5.2	a	EpRA-1900	*Mean Epoch of observed RA
96- 99	F4.1	ms	e_RAs	*Mean error in RA
101-105	F5.1	ms/ha	e_pmRA	Mean error in pmRA
107-111	F5.2	a	EpDE-1900	*Mean Epoch of observed DE
113-116	F4.1	carcsec	e_DEs	*Mean error in Declination
118-122	F5.1	carcsec/ha	e_pmDE	Mean error in pmDE
124-128	F5.2	mag	Vmag	*V magnitude
129	A1	---	n_Vmag	*[VvD] Magnitude flag
131-137	A7	---	SpType	*Spectral type(s)
139-144	F6.3	arcsec	plx	*?Parallax
147-152	F6.1	km/s	RV	*?Radial velocity
155-159	A5	---	AGK3R	AGK3R number (Catalog <I/72>)

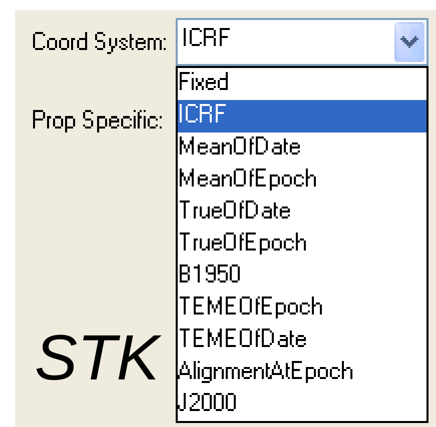
ICRF is the Current Realization of ICRS

Since 1998, IAU adopted the International Celestial Reference Frame (ICRF) as the standard reference frame: quasi-inertial reference frame with barely no time dependency.

It represents an improvement upon the theory behind the J2000 frame, and it is the best realization of an inertial frame constructed to date.

Very Long Baseline Interferometry





Further Reading on the Web Site

THE ASTRONOMICAL JOURNAL, 116:516–546, 1998 July

THE INTERNATIONAL CELESTIAL REFERENCE FRAME AS REALIZED BY VERY LONG BASELINE INTERFEROMETRY

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Received 1997 December 1; revised 1998 March 19

TABLE 3

Coordinates of the 212 Defining Sources in the ICRF

		Note[c]								Epoch of Observation[d]				
Designation[a]	Source[b]	X	S	H	α (J2000.0)	δ (J2000.0)	σ_α (s)	σ_δ (arcsec)	$C_{\alpha\text{-}\delta}$	Mean	First	Last	N_{exp}[e]	N_{obs}[f]
ICRF J000557.1+382015......	0003+380	...	...		00 05 57.175409	38 20 15.14857	0.000041	0.00051	−0.041	49,087.0	48,720.9	49,554.8	2	41
ICRF J001031.0+105829......	0007+106	...	...		00 10 31.005888	10 58 29.50412	0.000032	0.00068	0.540	47,938.9	47,288.7	49,690.0	10	74
ICRF J001033.9+172418......	0007+171	...	...		00 10 33.990619	17 24 18.76135	0.000021	0.00035	−0.402	48,730.8	47,931.6	49,662.8	19	57
ICRF J001331.1+405137......	0010+405	2	1		00 13 31.130213	40 51 37.14407	0.000026	0.00034	−0.038	49,549.6	48,434.7	49,820.5	7	219
ICRF J001708.4+813508......	0014+813	...	...		00 17 08.474953	81 35 08.13633	0.000121	0.00026	0.012	49,505.2	47,023.7	49,924.8	78	1453
ICRF J004204.5+232001......	0039+230	...	...		00 42 04.545183	23 20 01.06129	0.000036	0.00060	0.090	48,898.1	48,328.5	49,533.8	3	44
ICRF J004959.4−573827......	0047−579	...	...		00 49 59.473091	−57 38 27.33992	0.000047	0.00053	0.298	48,697.0	47,626.5	49,407.6	13	46
ICRF J011205.8+224438......	0109+224	...	...	Y	01 12 05.824718	22 44 38.78619	0.000027	0.00049	0.082	48,733.1	48,434.7	49,736.9	7	97
ICRF J012642.7+255901......	0123+257	...	...		01 26 42.792631	25 59 01.30079	0.000030	0.00054	0.167	48,856.4	48,328.5	49,659.8	4	71
ICRF J013305.7−520003......	0131−522	...	...		01 33 05.762585	−52 00 03.94693	0.000049	0.00081	0.399	49,039.1	48,162.4	49,895.6	6	30
ICRF J013658.5+475129......	0133+476	2	2		01 36 58.594810	47 51 29.10006	0.000026	0.00027	0.021	48,629.0	45,138.8	49,750.8	190	2196
ICRF J013738.3−243053......	0135−247	...	...		01 37 38.346378	−24 30 53.88526	0.000055	0.00042	−0.188	48,321.8	47,640.2	49,790.7	3	29
ICRF J014125.8−092843......	0138−097	2	1		01 41 25.832025	−09 28 43.67381	0.000081	0.00088	0.063	47,138.1	46,875.8	49,498.8	2	20
ICRF J015127.1+274441......	0148+274	...	...		01 51 27.146149	27 44 41.79365	0.000031	0.00043	−0.064	48,963.9	48,328.5	49,659.8	5	112
ICRF J015218.0+220707......	0149+218	...	...		01 52 18.059047	22 07 07.70004	0.000020	0.00029	−0.437	48,294.0	46,977.9	49,848.8	50	243
ICRF J015734.9+744243......	0153+744	4	3	Y	01 57 34.964908	74 42 43.22998	0.000091	0.00031	0.059	49,495.7	47,019.9	49,820.5	11	400
ICRF J020333.3+723253......	0159+723	...	...		02 03 33.385004	72 32 53.66741	0.000072	0.00031	0.033	48,800.7	47,011.4	49,667.9	17	108
ICRF J020504.9+321230......	0202+319	...	...		02 05 04.925371	32 12 30.09560	0.000022	0.00030	−0.441	48,017.7	45,466.3	49,736.9	35	214
ICRF J021748.9+014449......	0215+015	1	1		02 17 48.954740	01 44 49.69909	0.000022	0.00039	−0.215	49,302.1	48,328.5	49,547.8	5	133
ICRF J022239.6+430207......	0219+428	...	...		02 22 39.611500	43 02 07.79884	0.000034	0.00043	−0.098	49,103.6	48,650.8	49,554.8	7	64
ICRF J022256.4−344128......	0220−349	...	...		02 22 56.401625	−34 41 28.73011	0.000050	0.00044	−0.209	48,679.5	47,640.2	49,790.7	4	35
ICRF J022850.0+672103......	0224+671	...	...		02 28 50.051459	67 21 03.02926	0.000052	0.00031	−0.080	45,097.6	44,090.5	49,600.3	42	801
ICRF J022934.9−784745......	0230−790	...	...		02 29 34.946647	−78 47 45.60129	0.000149	0.00049	0.028	48,828.1	47,626.5	49,895.6	11	52
ICRF J023838.9+163659......	0235+164	1	1		02 38 38.930108	16 36 59.27471	0.000018	0.00027	0.090	47,475.7	44,447.0	49,909.6	194	2595
ICRF J024229.1+110100......	0239+108	2	2		02 42 29.170847	11 01 00.72823	0.000018	0.00030	−0.483	48,582.3	47,511.1	49,662.8	43	153
ICRF J025134.5+431515......	0248+430	...	...		02 51 34.536779	43 15 15.82858	0.000027	0.00033	−0.074	49,109.4	47,931.6	49,690.0	10	169
ICRF J025927.0+074739......	0256+075	...	...		02 59 27.076633	07 47 39.64323	0.000021	0.00035	−0.607	48,247.0	47,011.4	49,445.6	44	190
ICRF J030350.6−621125......	0302−623	...	...		03 03 50.631333	−62 11 25.54983	0.000047	0.00033	0.129	49,059.2	48,162.4	49,650.8	15	97
ICRF J030903.6+102916......	0306+102	...	...		03 09 03.623523	10 29 16.34082	0.000023	0.00042	−0.804	48,974.1	47,394.1	49,667.9	18	76
ICRF J030956.0−605839......	0308−611	...	...		03 09 56.099167	−60 58 39.05628	0.000038	0.00029	0.037	49,029.5	47,626.5	49,895.6	79	738
ICRF J031301.9+412001......	0309+411	...	...	Y	03 13 01.962129	41 20 01.18353	0.000026	0.00031	−0.321	48,371.0	47,165.8	49,848.8	29	127
ICRF J034506.4+145349......	0342+147	...	...		03 45 06.416546	14 53 49.55818	0.000021	0.00032	−0.622	48,809.6	47,394.1	49,445.6	23	177
ICRF J040305.5+260001......	0400+258	3	2	Y	04 03 05.586048	26 00 01.50274	0.000020	0.00030	−0.127	48,990.5	47,005.8	49,820.5	37	397
ICRF J040922.0+121739......	0406+121	2	1		04 09 22.008740	12 17 39.84750	0.000021	0.00033	−0.704	48,399.2	46,977.9	49,565.9	28	149
ICRF J041636.5−185108......	0414−189	...	...		04 16 36.544466	−18 51 08.34012	0.000051	0.00048	−0.078	47,814.6	46,840.8	49,790.7	3	31
ICRF J042442.2−375620......	0422−380	...	...		04 24 42.243727	−37 56 20.78423	0.000033	0.00119	0.251	49,081.7	48,162.4	49,750.8	11	60
ICRF J042446.8+003606......	0422+004	2	1		04 24 46.842052	00 36 06.32983	0.000020	0.00063	0.038	48,938.2	45,997.8	49,820.5	11	245
ICRF J042636.6+051819......	0423+051	...	...		04 26 36.604102	05 18 19.87204	0.000031	0.00087	0.101	48,977.3	48,194.7	49,667.9	9	64
ICRF J042840.4−375619......	0426−380	...	...		04 28 40.424306	−37 56 19.58031	0.000036	0.00047	0.011	48,125.7	47,640.2	49,692.6	5	39
ICRF J043900.8−452222......	0437−454	...	...		04 39 00.854714	−45 22 22.56260	0.000057	0.00078	−0.123	49,443.5	48,766.9	49,895.6	7	32
ICRF J044238.6−001743......	0440−003	1	1		04 42 38.660762	−00 17 43.41910	0.000025	0.00064	0.262	47,735.2	47,011.4	49,576.9	15	111
ICRF J044907.6+112128......	0446+112	...	...		04 49 07.671119	11 21 28.59662	0.000024	0.00051	−0.143	49,312.0	47,394.1	49,854.8	5	32
ICRF J045005.4−810102......	0454−810	...	...		04 50 05.440195	−81 01 02.23146	0.000137	0.00032	−0.005	48,784.2	47,626.5	49,895.6	18	148
ICRF J045952.0+022931......	0457+024	...	...		04 59 52.050664	02 29 31.17631	0.000019	0.00032	0.062	48,993.4	47,005.8	49,750.8	36	394
ICRF J050145.2+135607......	0458+138	2	2		05 01 45.270840	13 56 07.22063	0.000037	0.00064	−0.770	48,830.7	47,394.1	49,848.8	13	20
ICRF J050523.1+045942......	0502+049	...	...		05 05 23.184723	04 59 42.72448	0.000037	0.00060	−0.584	48,897.7	47,394.1	49,667.9	6	28
ICRF J050643.9−610940......	0506−612	...	...		05 06 43.988739	−61 09 40.99328	0.000047	0.00035	0.145	48,760.5	48,110.9	49,594.7	16	69
ICRF J050842.3+843204......	0454+844	...	...		05 08 42.363503	84 32 04.54402	0.000194	0.00028	−0.046	48,674.7	46,977.9	49,611.9	42	250
ICRF J051002.3+180041......	0507+179	2	2		05 10 02.369122	18 00 41.58171	0.000020	0.00030	−0.396	49,401.9	47,605.1	49,820.5	24	339
ICRF J051644.9−620705......	0516−621	...	...		05 16 44.926178	−62 07 05.38930	0.000048	0.00042	0.202	49,455.4	48,749.6	49,895.6	9	56

Formal Definition of ICRS

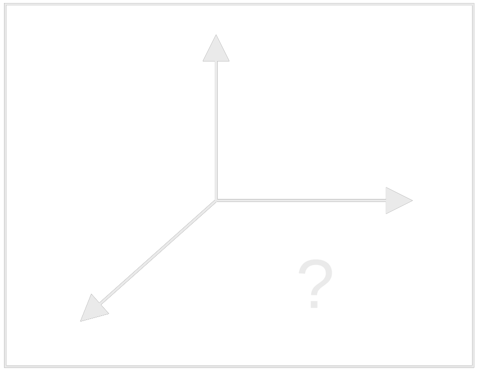
It is defined by the measured positions of 212 extragalactic sources (mainly quasars).

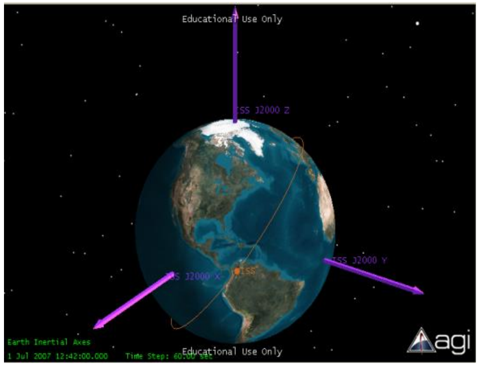
1. Its **origin** is located at the barycenter of the solar system through appropriate modeling of VLBI observations in the framework of general relativity.

2. Its **pole** is in the direction defined by the conventional IAU models for precession (Lieske et al. 1977) and nutation (Seidelmann 1982).

3. Its **origin of right ascensions** was implicitly defined by fixing the right ascension of the radio source 3C273B to FK5 J2000 value.

3. The Orbit in Space





Coordinate systems

Coordinate Systems

Now that we have defined an inertial reference frame, other reference frames can be defined according to the needs of the considered application.

Coordinate transformations between two reference frames involve rotation and translation.

What are the possibilities for a satellite in Earth orbit ?

For Your Project

1. Geocentric inertial frame in which you express the governing equations of motion:

   The geocentric celestial reference frame (GCRF/ECI) is the counterpart of the ICRF and is the standard inertial coordinate system for the Earth.

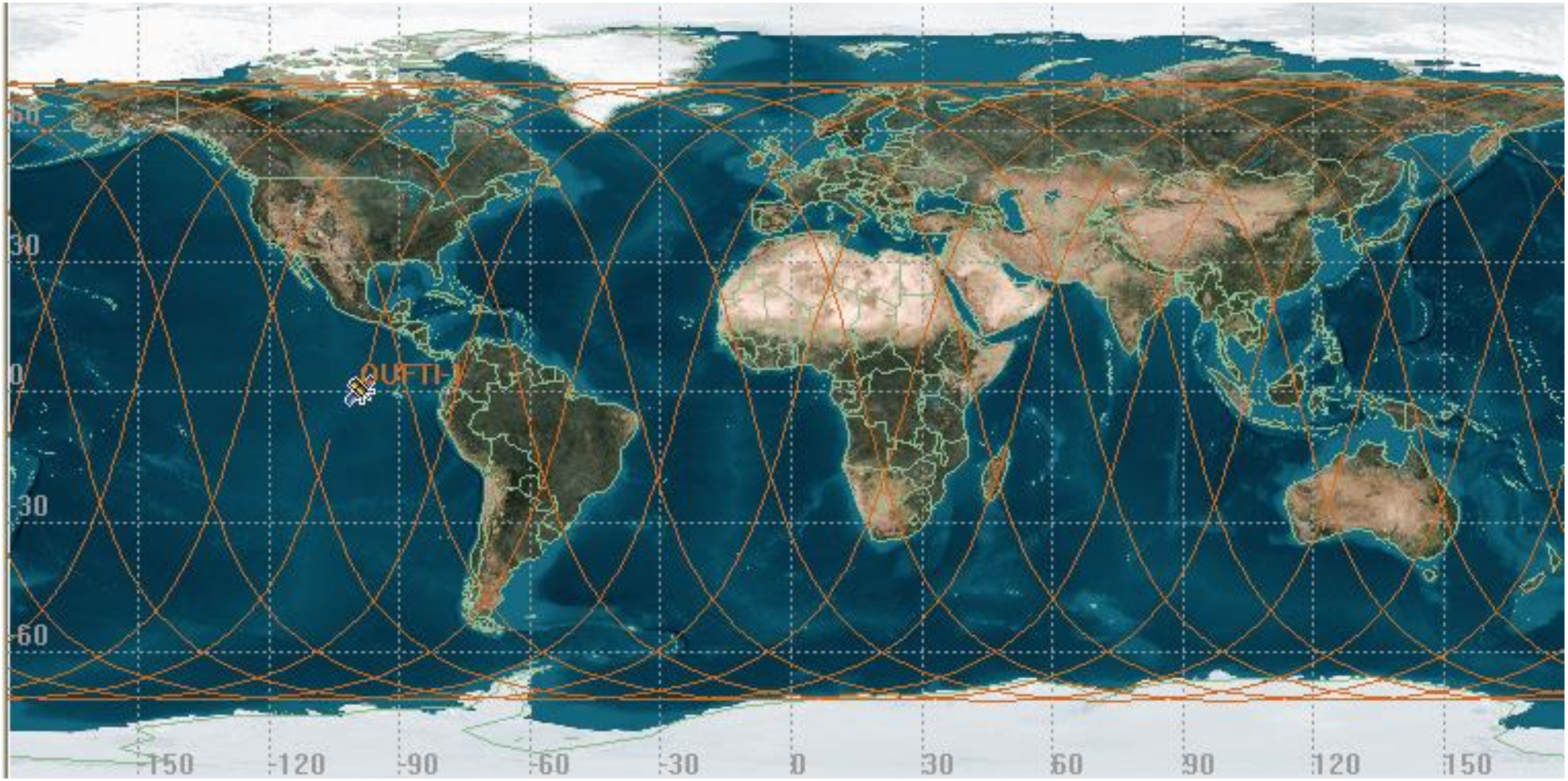
2. Geocentric frame (ITRF/ECEF) rotating with the Earth to calculate the gravitational force and for ground tracks

   ⇒ z-axis is parallel to Earth's rotation vector.

   ⇒ x-axis passes through the Greenwich meridian.

   ⇒ y-axis: right-handed set.

Example of a Ground Track



Simplified ECEF-ECI Transformation

$$\omega_{\oplus} = 0.000{,}072{,}921{,}158{,}553{,}0 \text{ rad/s}$$

$$\theta_{\text{GMST,2000}} = 1.7447671633061 \text{ rad}$$

$$\theta_{\text{GMST}} = \theta_{\text{GMST,2000}} + \omega_{\oplus} \times 86400 \times (t + 0.5) \text{ rad}$$

$$\begin{pmatrix} \mathrm{x} \\ \mathrm{y} \\ \mathrm{z} \end{pmatrix}_{ECI} = \begin{bmatrix} \cos(\theta_{\text{GMST}}) & -\sin(\theta_{\text{GMST}}) & 0 \\ \sin(\theta_{\text{GMST}}) & \cos(\theta_{\text{GMST}}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{ECEF}$$

Precession, nutation, polar motion ignored

The Complete/Accurate ECEF-ECI Transformation

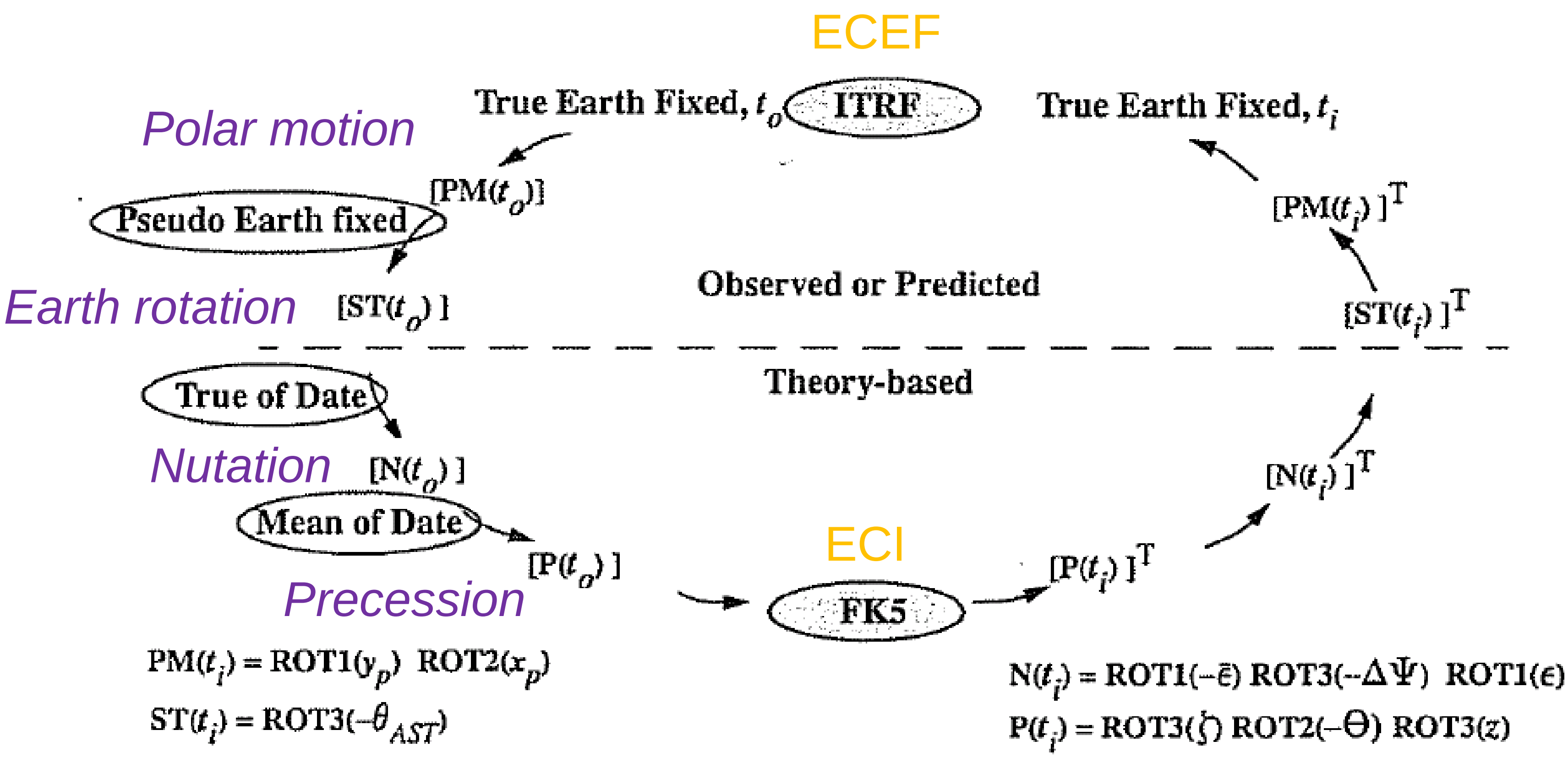
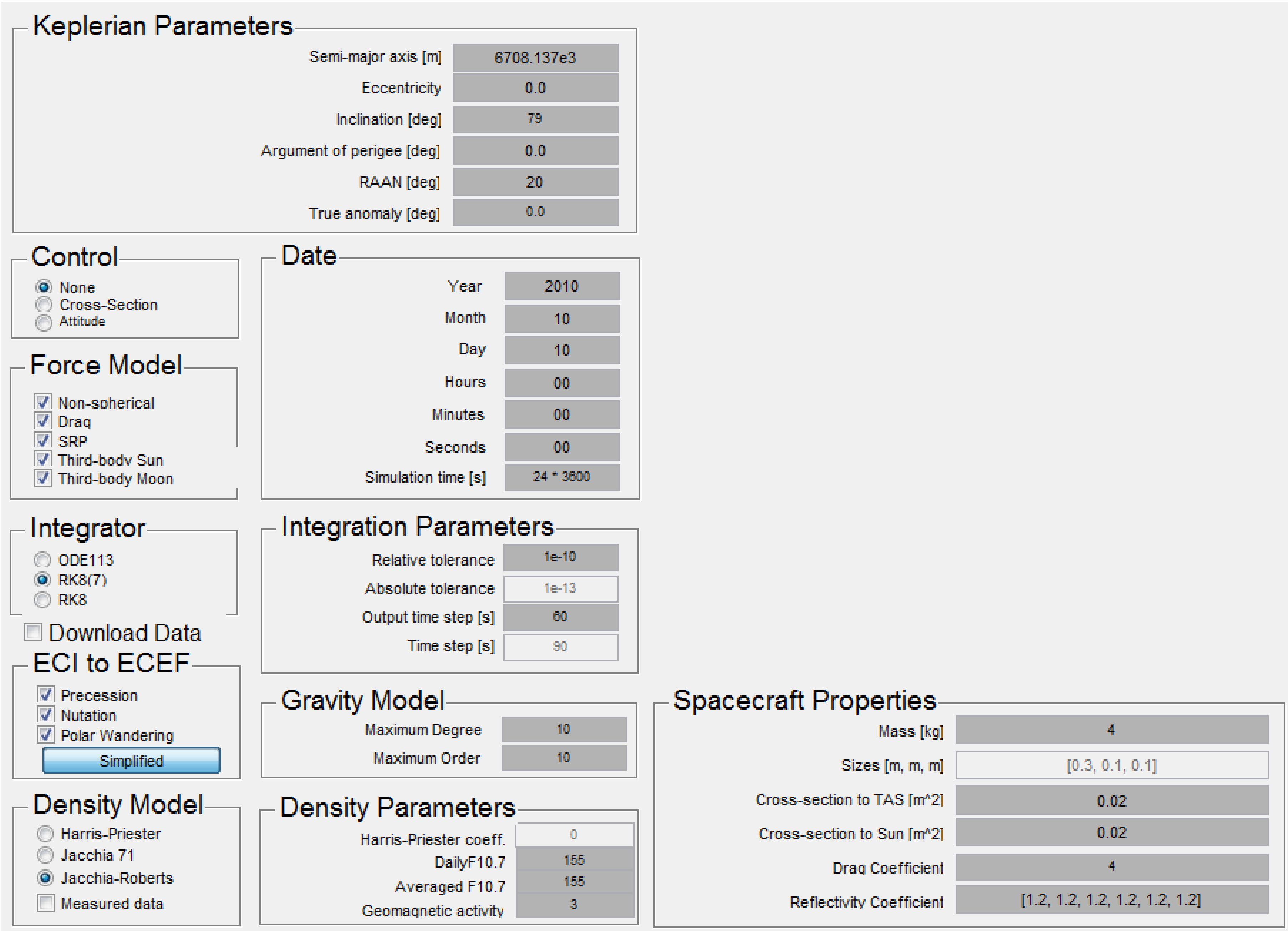


Figure 3-29. Classical Transformation. This figure depicts the transformation of a state vector in the body fixed (ITRF) frame to the inertial (FK5) frame. This two-way conversion is necessary for many orbit determination problems. The clear ellipses show the intermediate frames.

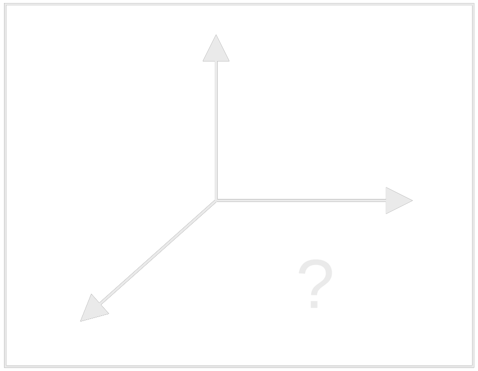
Vallado, *Fundamental of Astrodynamics and Applications*, Kluwer, 2001.

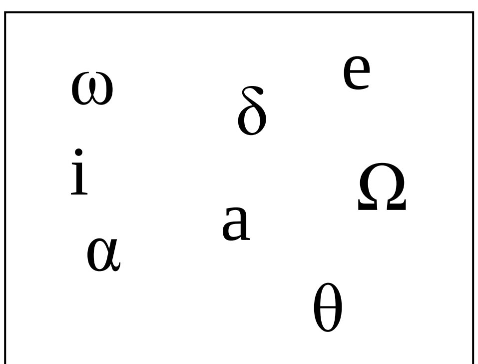

Keplerian Parameters
Semi-major axis [m]
6708.137e3
Eccentricity
0.0
Inclination [deg]
79
Argument of perigee [deg]
0.0
RAAN [deg]
20
True anomaly [deg]
0.0
Control
None
Cross-Section
Attitude
Date
Year
2010
Month
10
Day
10
Hours
00
Minutes
00
Seconds
00
Simulation time [s]
24 * 3600
Force Model
Non-spherical
Drag
SRP
Third-body Sun
Third-body Moon
Integrator
ODE113
RK8(7)
RK8
Integration Parameters
Relative tolerance
1e-10
Absolute tolerance
1e-13
Output time step [s]
60
Time step [s]
90
Download Data
ECI to ECEF
Precession
Nutation
Polar Wandering
Simplified
Gravity Model
Maximum Degree
10
Maximum Order
10
Spacecraft Properties
Mass [kg]
4
Sizes [m, m, m]
[0.3, 0.1, 0.1]
Cross-section to TAS [m^2]
0.02
Cross-section to Sun [m^2]
0.02
Drag Coefficient
4
Reflectivity Coefficient
[1.2, 1.2, 1.2, 1.2, 1.2, 1.2]
Density Model
Harris-Priester
Jacchia 71
Jacchia-Roberts
Measured data
Density Parameters
Harris-Priester coeff.
0
DailyF10.7
155
Averaged F10.7
155
Geomagnetic activity
3

Yet More Coordinate Systems !

Satellite coordinate system	**For ADCS**
Perifocal coordinate system	**Natural frame for an orbit (z is zero)**
Heliocentric coordinate system	**For interplanetary missions**
Non-singular elements	**For particular orbits**

3. The Orbit in Space





Coordinate types

Different Coordinates

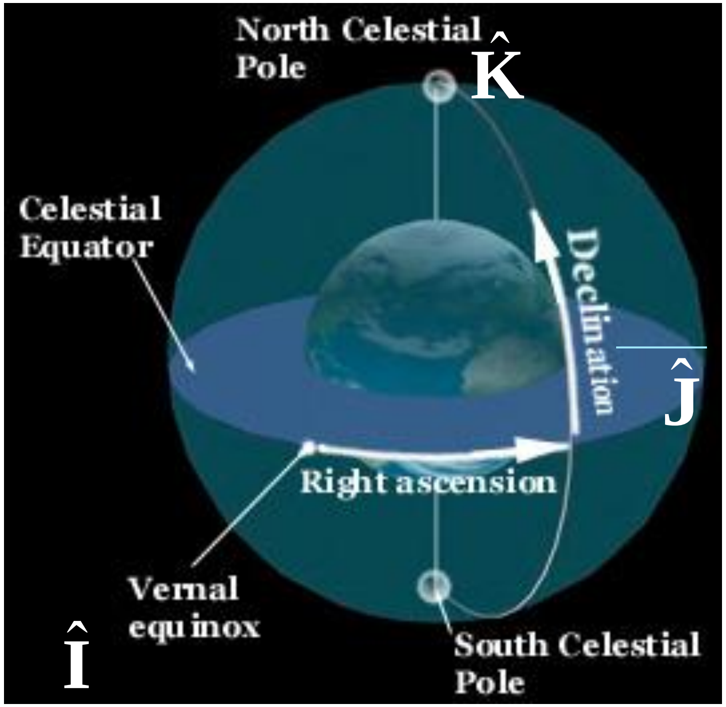
1. Cartesian: **for computations**

2. Keplerian elements: **for physical interpretation**

3. Two-line elements: **for downloading satellite data**

4. Spherical: azimuth and elevation (**for ground station**) — right ascension and declination (**for astronomers**)

Inertial Frame (I,J,K) for Cartesian Coordinates



Orbital (Keplerian) Elements

For interpretation

r and **v** do not directly yield much information about the orbit. We cannot even infer from them what type of conic the orbit represents !

Another set of six variables, which is much more descriptive of the orbit, is needed.

6 Orbital (Keplerian) Elements

1. *e*: shape of the orbit
2. *a*: size of the orbit

definition of the ellipse

3. *i*: orients the orbital plane with respect to the ecliptic plane
4. Ω: longitude of the intersection of the orbital and ecliptic planes

definition of the orbital plane

5. ω: orients the semi-major axis with respect to the ascending node

orientation of the ellipse within the orbital plane

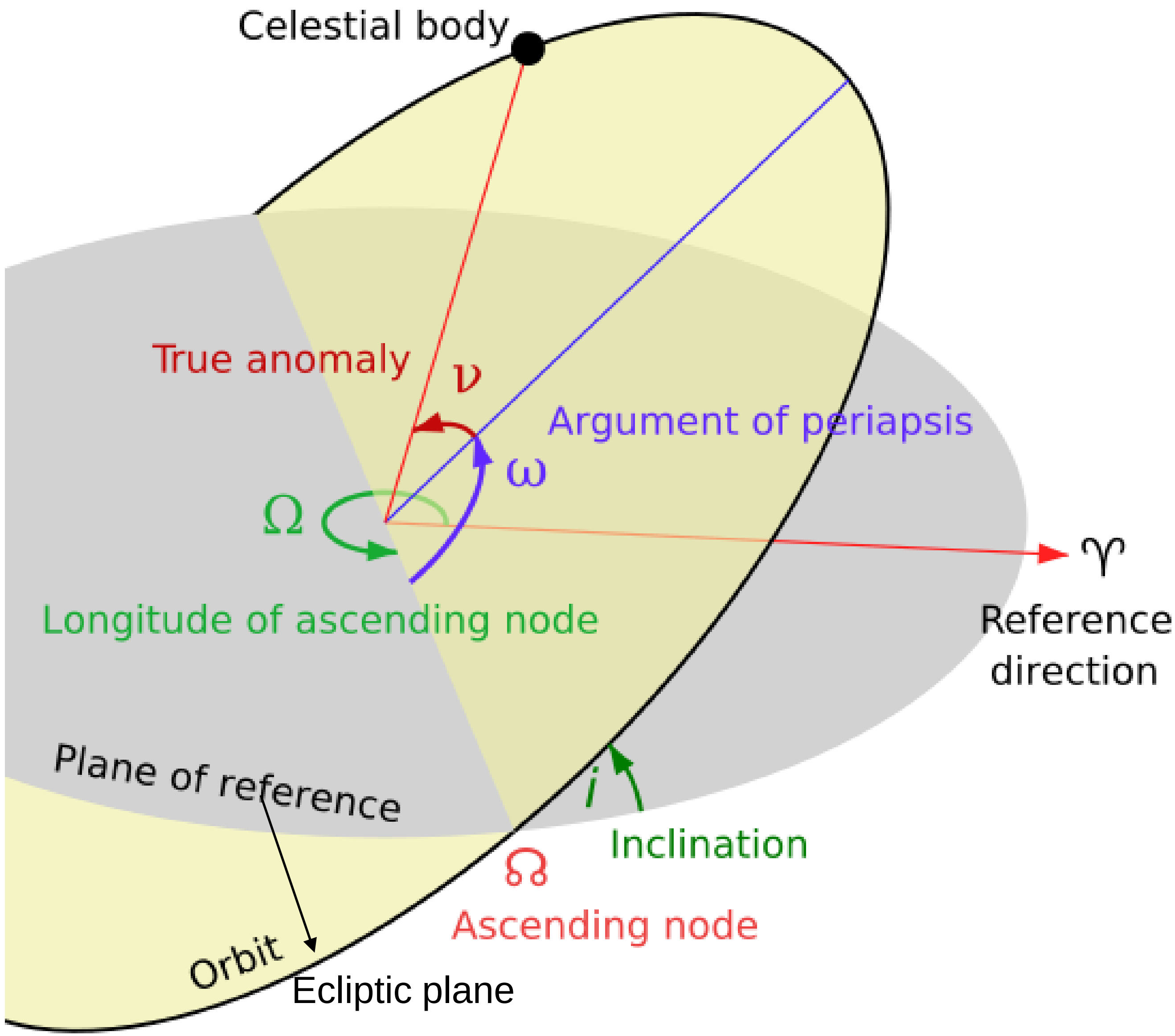
6. v: orients the celestial body in space

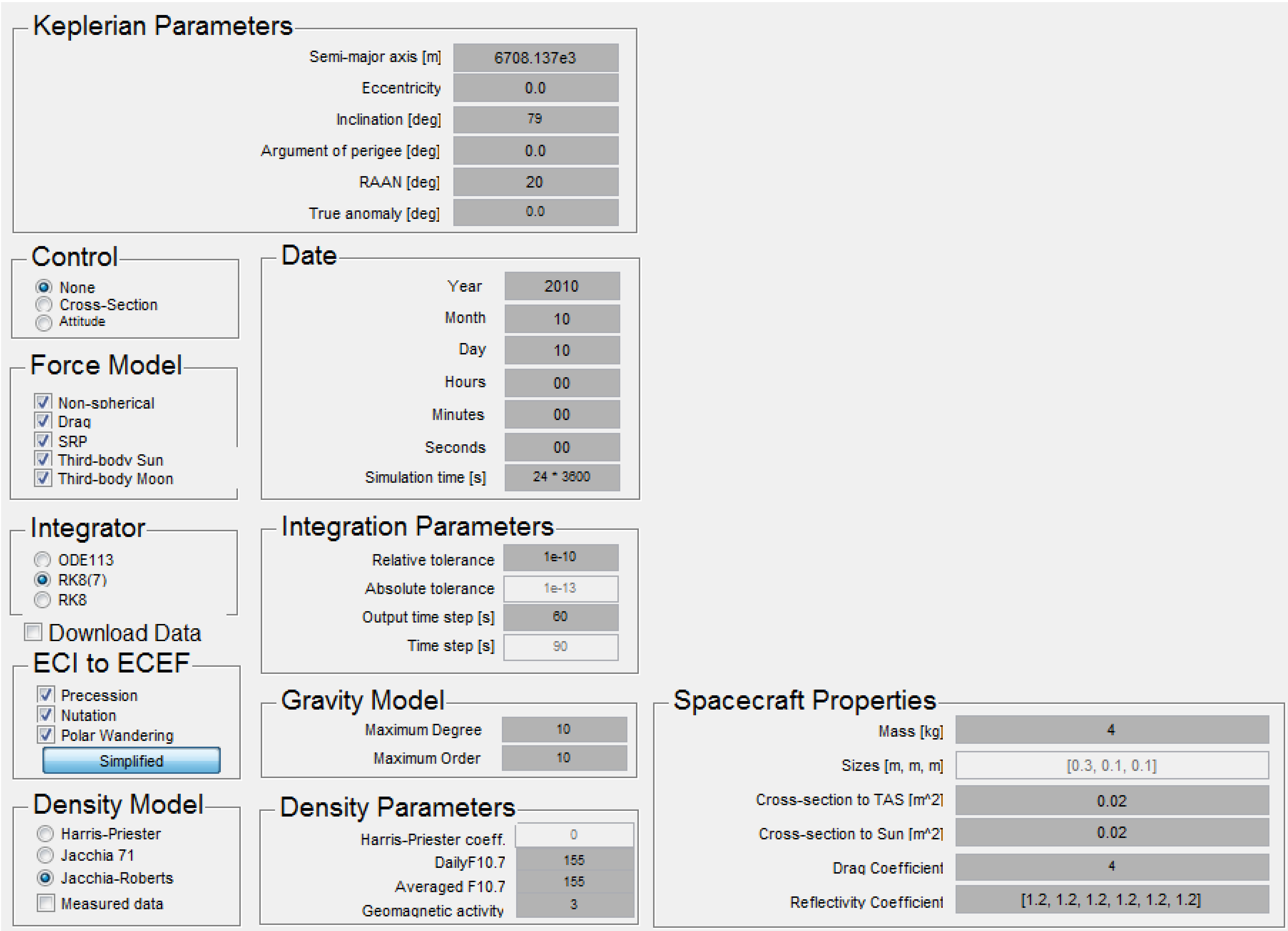
position of the satellite on the ellipse

Orbital plane

orientation of the ellipse

position of the satellite

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Semi-major axis [m]
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Inclination [deg]
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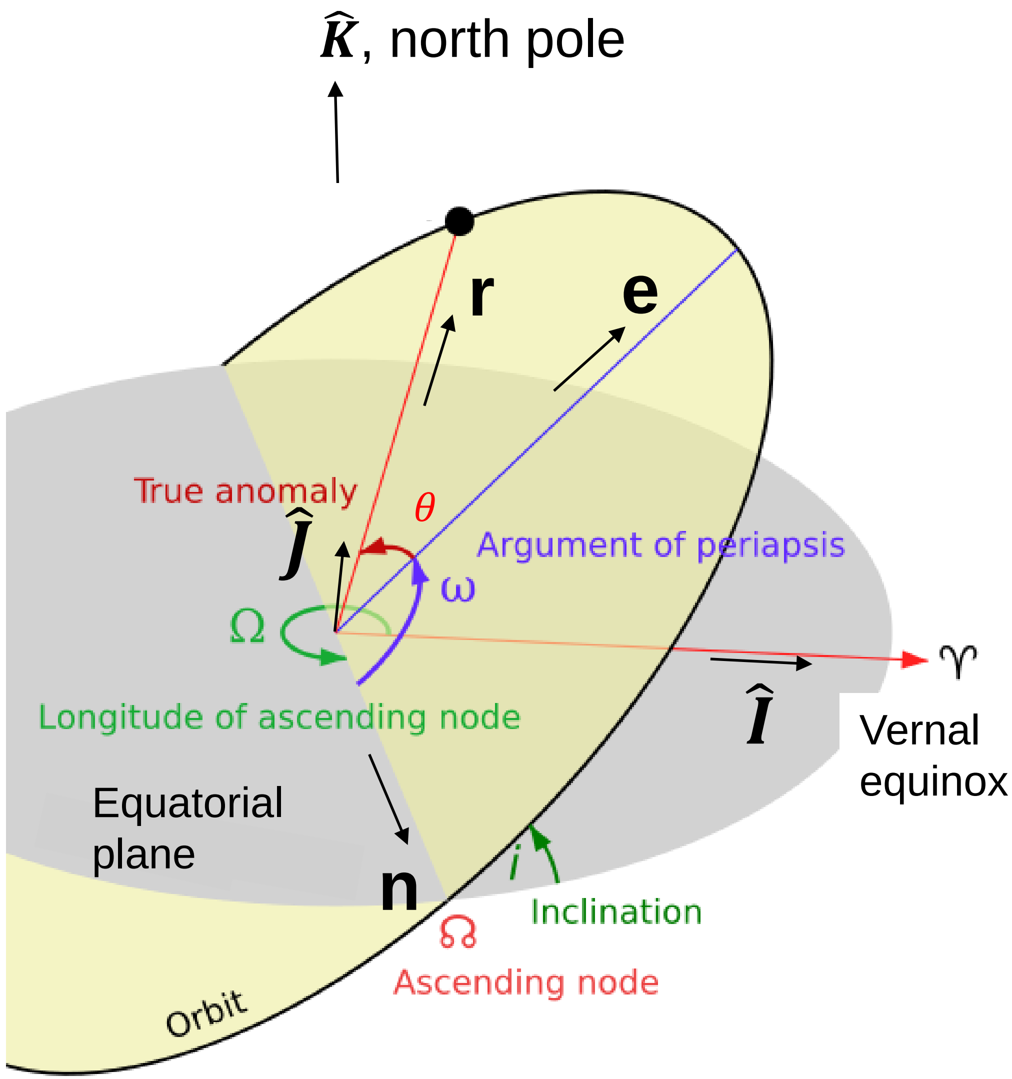
Argument of perigee [deg]
0.0
RAAN [deg]
20
True anomaly [deg]
0.0
Control
None
Cross-Section
Attitude
Date
Year
2010
Month
10
Day
10
Hours
00
Minutes
00
Seconds
00
Simulation time [s]
24 * 3600
Force Model
Non-spherical
Drag
SRP
Third-body Sun
Third-body Moon
Integrator
ODE113
RK8(7)
RK8
Integration Parameters
Relative tolerance
1e-10
Absolute tolerance
1e-13
Output time step [s]
60
Time step [s]
90
Download Data
ECI to ECEF
Precession
Nutation
Polar Wandering
Simplified
Gravity Model
Maximum Degree
10
Maximum Order
10
Spacecraft Properties
Mass [kg]
4
Sizes [m, m, m]
[0.3, 0.1, 0.1]
Cross-section to TAS [m^2]
0.02
Cross-section to Sun [m^2]
0.02
Drag Coefficient
4
Reflectivity Coefficient
[1.2, 1.2, 1.2, 1.2, 1.2, 1.2]
Density Model
Harris-Priester
Jacchia 71
Jacchia-Roberts
Measured data
Density Parameters
Harris-Priester coeff.
0
DailyF10.7
155
Averaged F10.7
155
Geomagnetic activity
3

Orbital Elements a,e,i,Ω,ω,θ from r,v ?



e and a from the 2-body Problem

$$\mu \boldsymbol{e} = \boldsymbol{v} \times \boldsymbol{h} - \mu \frac{\boldsymbol{r}}{r}$$

$$v = \sqrt{\mu \left(\frac{2}{r} - \frac{1}{a} \right)}$$

⇩

$$e = \left\| \frac{\boldsymbol{v} \times (\boldsymbol{r} \times \boldsymbol{v})}{\mu} - \frac{\boldsymbol{r}}{r} \right\|$$

⇩

$$a = \frac{r}{2 - \frac{r v^2}{\mu}}$$

$$r = \|\boldsymbol{r}\|, v = \|\boldsymbol{v}\|$$

Inclination

Angle between the orbital and equatorial planes:

Normal to the orbit plane → $\boldsymbol{h}$

Normal to the equatorial plane → $\widehat{\boldsymbol{K}}$

$$\cos i = \frac{\boldsymbol{h}.\widehat{\boldsymbol{K}}}{\|\boldsymbol{h}\|}$$

$$\boldsymbol{h} = \boldsymbol{r} \times \boldsymbol{v}$$

$$i = \cos^{-1}\left(\frac{(\boldsymbol{r} \times \boldsymbol{v}).\widehat{\boldsymbol{K}}}{\|\boldsymbol{r} \times \boldsymbol{v}\|}\right)$$

Longitude Ω

Angle between the nodal vector **n** and the vernal equinox:

$$\cos\Omega = \frac{\boldsymbol{n}.\hat{\boldsymbol{I}}}{\|\boldsymbol{n}\|}$$

The nodal vector **n** is in the orbital and equatorial planes:

$$\boldsymbol{n} = \hat{\boldsymbol{K}} \times \frac{\boldsymbol{h}}{h}$$

$$\Omega = cos^{-1}\frac{\boldsymbol{n}.\hat{\boldsymbol{I}}}{\|\boldsymbol{n}\|} = cos^{-1}\left(\frac{\left(\hat{\boldsymbol{K}} \times \frac{\boldsymbol{r}\times\boldsymbol{v}}{\|\boldsymbol{r}\times\boldsymbol{v}\|}\right).\hat{\boldsymbol{I}}}{\left\|\hat{\boldsymbol{K}} \times \frac{\boldsymbol{r}\times\boldsymbol{v}}{\|\boldsymbol{r}\times\boldsymbol{v}\|}\right\|}\right) \qquad \boldsymbol{n}.\hat{\boldsymbol{J}} \geq \boldsymbol{0}$$

$$\Omega = 360° - \Omega \qquad \boldsymbol{n}.\hat{\boldsymbol{J}} < \boldsymbol{0}$$

Argument of Perigee

Angle between the nodal and eccentricity vectors:

$$\cos\omega = \frac{\boldsymbol{e}.\boldsymbol{n}}{\|\boldsymbol{e}\|\|\boldsymbol{n}\|}$$

$$\boldsymbol{n} = \widehat{\boldsymbol{K}} \times \frac{\boldsymbol{h}}{h}, \boldsymbol{e} = \frac{\boldsymbol{v} \times (\boldsymbol{r} \times \boldsymbol{v})}{\mu} - \frac{\boldsymbol{r}}{r}$$

$$\omega = cos^{-1}\left(\frac{\left(\widehat{\boldsymbol{K}} \times \frac{\boldsymbol{r} \times \boldsymbol{v}}{\|\boldsymbol{r} \times \boldsymbol{v}\|}\right).\left(\frac{\boldsymbol{v} \times (\boldsymbol{r} \times \boldsymbol{v})}{\mu} - \frac{\boldsymbol{r}}{r}\right)}{\left\|\widehat{\boldsymbol{K}} \times \frac{\boldsymbol{r} \times \boldsymbol{v}}{\|\boldsymbol{r} \times \boldsymbol{v}\|}\right\|\left\|\frac{\boldsymbol{v} \times (\boldsymbol{r} \times \boldsymbol{v})}{\mu} - \frac{\boldsymbol{r}}{r}\right\|}\right) \qquad \boldsymbol{e}.\widehat{\boldsymbol{K}} \geq \boldsymbol{0}$$

$$\omega = 360° - \omega \qquad \boldsymbol{e}.\widehat{\boldsymbol{K}} < \boldsymbol{0}$$

True Anomaly

Angle between the position and eccentricity vectors

$$\cos\theta = \frac{\boldsymbol{r}.\boldsymbol{e}}{r\|\boldsymbol{e}\|}$$

$$\theta = cos^{-1}\left(\frac{\boldsymbol{r}.\left(\frac{\boldsymbol{v}\times(\boldsymbol{r}\times\boldsymbol{v})}{\mu}-\frac{\boldsymbol{r}}{r}\right)}{r\left\|\frac{\boldsymbol{v}\times(\boldsymbol{r}\times\boldsymbol{v})}{\mu}-\frac{\boldsymbol{r}}{r}\right\|}\right) \qquad \boldsymbol{r}.\boldsymbol{v}\geq\boldsymbol{0}$$

$$\theta = 360° - \theta \qquad \boldsymbol{r}.\boldsymbol{v}<\boldsymbol{0}$$

r,v from a,e,i,Ω,ω,θ ? From Vallado

2.6 Application: *r* and *v* from Orbital Elements

We've seen how to find the orbital elements from the position and velocity vectors, but we often need the reverse process to complete certain astrodynamic studies. We'll call the process ***RANDV*** to indicate that we're determining the position and velocity vectors. The overall idea is to determine the position and velocity vectors in the perifocal coordinate system, PQW, and then rotate to the geocentric equatorial system. Although the orbit may not be elliptical, and therefore the PQW system would actually be undefined, we can elegantly work around this limitation. We can also make the method completely generic through several short, simple substitutions.

First, we must use the semiparameter instead of the semimajor axis. As previously mentioned, the semimajor axis is infinite for the parabola, whereas the semiparameter is defined for all orbits. The second requirement concerns how we treat the auxiliary classical orbital elements for the special cases of circular and equatorial orbits.

Let's begin by finding the position and velocity vectors in the perifocal coordinate system. We've developed and presented these equations previously but show them here coupled with the trajectory equation. Notice the use of the semiparameter to replace dependence on the semimajor axis.

$$\vec{r}_{PQW} = \begin{bmatrix} \frac{p\cos(\nu)}{1 + e\cos(\nu)} \\ \frac{p\sin(\nu)}{1 + e\cos(\nu)} \\ 0 \end{bmatrix} \tag{2-100}$$

An immediate difficulty arises when attempting to define the true anomaly for circular orbits. It turns out that the orbital elements may be *temporarily* replaced with the alternate elements to provide the necessary values for the calculations. Although you can design a change like this so it's transparent to users, make sure any changes or alternate codings use temporary variables and don't alter the original elements. It's possible to substitute values:

$$\begin{gathered} \text{IF Circular Equatorial} \\ \text{let } \omega = 0.0, \Omega = 0.0, \text{ and } \nu = \lambda_{true} \\ \text{IF Circular Inclined} \\ \text{let } \omega = 0.0 \text{ and } \nu = u \end{gathered} \tag{2-101}$$

The rationale for assigning ω and Ω to zero will be clear shortly; however, we haven't violated any assumptions because ω and Ω are undefined for circular orbits. Be careful not to return any changed variables in computer applications.

Find the velocity vector by differentiating the position vector:

$$\vec{v}_{PQW} = \begin{bmatrix} \dot{r}\cos(\nu) - r\dot{\nu}\sin(\nu) \\ \dot{r}\sin(\nu) + r\dot{\nu}\cos(\nu) \\ 0 \end{bmatrix}$$

Remembering the geometry from Fig. 1-13, solve Eq. (1-18) as

$$r\dot{\nu} = \frac{h}{r}$$

Now, substitute the definitions of position and angular momentum:

$$r\dot{\nu} = \frac{\sqrt{\mu p}(1 + e\,\mathrm{COS}(\nu))}{p} = \sqrt{\frac{\mu}{p}}(1 + e\,\mathrm{COS}(\nu))$$

Using Eq. (1-25) and the equation above, write

$$\dot{r} = \sqrt{\frac{\mu}{p}}(e\,\mathrm{SIN}(\nu))$$

Substituting these results into the differentiated vector gives us the final solution:

$$\vec{v}_{PQW} = \begin{bmatrix} -\sqrt{\frac{\mu}{p}}\,\mathrm{SIN}(\nu) \\ \sqrt{\frac{\mu}{p}}(e + \mathrm{COS}(\nu)) \\ 0 \end{bmatrix} \qquad (2\text{-}102)$$

The next step is to rotate the position and velocity vectors to the geocentric equatorial frame. Although this is relatively easy for standard, elliptical, inclined orbits, we'll need to take certain precautions in order to account for special cases, as described with the true anomaly above. We've discussed two of these special cases; the third is the elliptical equatorial case:

$$\begin{array}{c} \text{IF Elliptical Equatorial} \\ \text{set } \Omega = 0.0 \text{ and } \omega = \tilde{\omega}_{true} \end{array} \qquad (2\text{-}103)$$

The assumptions remain intact because Ω is undefined for elliptical equatorial orbits.

We can now do the coordinate transformations using Eq. (3-28). We may want to multiply out these operations to reduce trigonometric operations. The rationale for setting certain variables to zero should now be apparent. For the special cases, a zero rotation causes the vector to remain unchanged, whereas a desired angular value causes a change.

Implementing RANDV

Computational efficiency results from assigning the trigonometric terms [$\mathrm{SIN}(\nu)$, $\mathrm{COS}(\nu)$] and (μ/p) to temporary variables. This saves *many* transcendental operations and requires very little extra work. There are also some savings in treating special-case orbits if we reuse the same rotation matrices, but there may be some redundancy in special cases.

As with the ***ELORB*** algorithm, we may run many test cases to verify the routine. Because ***RANDV*** is simply designed to be a mirror calculation of the ***ELORB*** routine, we can use the same set of test reference data. But we must test several limiting cases. Algorithm 10 summarizes the process.

ALGORITHM 10: ***RANDV*** ($p, e, i, \Omega, \omega, \nu(u, \lambda_{true}, \tilde{\omega}_{true}) \Rightarrow \vec{r}_{IJK}\ \vec{v}_{IJK}$)

IF Circular Equatorial

SET $(\omega, \Omega) = 0.0$ and $\nu = \lambda_{true}$

IF Circular Inclined

SET $\omega = 0.0$ and $\nu = u$

IF Elliptical Equatorial

SET $\Omega = 0.0$ and $\omega = \tilde{\omega}_{true}$

$$\vec{r}_{PQW} = \begin{bmatrix} \dfrac{p\cos(\nu)}{1 + e\cos(\nu)} \\ \dfrac{p\sin(\nu)}{1 + e\cos(\nu)} \\ 0 \end{bmatrix} \qquad \vec{v}_{PQW} = \begin{bmatrix} -\sqrt{\dfrac{\mu}{p}}\sin(\nu) \\ \sqrt{\dfrac{\mu}{p}}(e + \cos(\nu)) \\ 0 \end{bmatrix}$$

$$\vec{r}_{IJK} = [\mathrm{ROT3}(-\Omega)][\mathrm{ROT1}(-i)][\mathrm{ROT3}(-\omega)]\vec{r}_{PQW} = [\tfrac{IJK}{PQW}]\vec{r}_{PQW}$$

$$\vec{v}_{IJK} = [\mathrm{ROT3}(-\Omega)][\mathrm{ROT1}(-i)][\mathrm{ROT3}(-\omega)]\vec{v}_{PQW} = [\tfrac{IJK}{PQW}]\vec{v}_{PQW}$$

$$\left[\frac{IJK}{PQW}\right] = \begin{bmatrix} \cos(\Omega)\cos(\omega) - \sin(\Omega)\sin(\omega)\cos(i) & -\cos(\Omega)\sin(\omega) - \sin(\Omega)\cos(\omega)\cos(i) & \sin(\Omega)\sin(i) \\ \sin(\Omega)\cos(\omega) + \cos(\Omega)\sin(\omega)\cos(i) & -\sin(\Omega)\sin(\omega) + \cos(\Omega)\cos(\omega)\cos(i) & -\cos(\Omega)\sin(i) \\ \sin(\omega)\sin(i) & \cos(\omega)\sin(i) & \cos(i) \end{bmatrix}$$

An example demonstrates the technique.

▼ **Example 2-6. Finding Position and Velocity Vectors (RANDV Test Case).**

GIVEN: $p = 11{,}067.790$ km $= 1.735\,27$ ER, $e = 0.832\,85$, $i = 87.87°$, $\Omega = 227.89°$, $\omega = 53.38°$, $\nu = 92.335°$

FIND: $\vec{r}_{IJK}\ \vec{v}_{IJK}$

We have to change the rotation angles if we're using special orbits (equatorial or circular), but this orbit doesn't have special cases. From the given information, form the PQW position and velocity vectors:

$$\vec{r}_{PQW} = \begin{bmatrix} \dfrac{p\cos(\nu)}{1 + e\cos(\nu)} \\ \dfrac{p\sin(\nu)}{1 + e\cos(\nu)} \\ 0 \end{bmatrix} = \begin{bmatrix} \dfrac{1.735\,27\cos(92.336)°}{1 + 0.832\,84\cos(92.336)°} \\ \dfrac{1.735\,27\sin(92.336)}{1 + 0.832\,84\cos(92.336)°} \\ 0 \end{bmatrix} = \begin{bmatrix} -0.073\,186\,7 \\ 1.794\,733\,9 \\ 0 \end{bmatrix} \text{ER}$$

$$\vec{v}_{PQW} = \begin{bmatrix} -\sqrt{\frac{\mu}{p}}\,\text{SIN}(\nu) \\ \sqrt{\frac{\mu}{p}}(e + \text{COS}(\nu)) \\ 0 \end{bmatrix} = \begin{bmatrix} -\sqrt{\frac{1}{1.735\,27}}\,\text{SIN}(92.336) \\ \sqrt{\frac{1}{1.735\,27}}(0.832\,84 + \text{COS}(92.336)) \\ 0 \end{bmatrix} = \begin{bmatrix} -0.758\,499\,8 \\ 0.601\,313\,6 \\ 0 \end{bmatrix} \frac{\text{ER}}{\text{TU}}$$

Rotate these vectors to the geocentric equatorial system using the following rotation matrices:

$$\vec{r}_{IJK} = [\text{ROT3}(-\Omega)][\text{ROT1}(-i)][\text{ROT3}(-\omega)]\vec{r}_{PQW}$$

$$\vec{v}_{IJK} = [\text{ROT3}(-\Omega)][\text{ROT1}(-i)][\text{ROT3}(-\omega)]\vec{v}_{PQW}$$

Or, use the expanded matrix with a computer to do the many trigonometric operations, which result in the transformation matrix

$$\left[\frac{IJK}{PQW}\right] = \begin{bmatrix} -0.377\,736\,47 & 0.554\,597\,39 & -0.741\,442\,44 \\ -0.462\,538\,21 & 0.580\,670\,14 & 0.669\,985\,52 \\ 0.802\,105\,71 & 0.596\,023\,42 & 0.037\,182\,20 \end{bmatrix}$$

Finally, multiply each vector to apply the transformation:

$$\vec{r}_{IJK} = \left[\frac{IJK}{PQW}\right]\vec{r}_{PQW} = \begin{bmatrix} -0.377\,736\,47 & 0.554\,597\,39 & -0.741\,442\,44 \\ -0.462\,538\,21 & 0.580\,670\,14 & 0.669\,985\,52 \\ 0.802\,105\,71 & 0.596\,023\,42 & 0.037\,182\,20 \end{bmatrix} \begin{bmatrix} -0.073\,186\,7 \\ 1.794\,733\,9 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1.023 \\ 1.076 \\ 1.011 \end{bmatrix} \text{ER} = \begin{bmatrix} 6524.834 \\ 6862.875 \\ 6448.296 \end{bmatrix} \text{km}$$

$$\vec{v}_{IJK} = \left[\frac{IJK}{PQW}\right]\vec{v}_{PQW} = \begin{bmatrix} -0.377\,736\,47 & 0.554\,597\,39 & -0.741\,442\,44 \\ -0.462\,538\,21 & 0.580\,670\,14 & 0.669\,985\,52 \\ 0.802\,105\,71 & 0.596\,023\,42 & 0.037\,182\,20 \end{bmatrix} \begin{bmatrix} -0.758\,499\,8 \\ 0.601\,313\,6 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.62 \\ 0.70 \\ 0.25 \end{bmatrix} \text{ER/TU} = \begin{bmatrix} 4.901\,320 \\ 5.533\,756 \\ -1.976\,341 \end{bmatrix} \text{km/s}$$

Two-Line Elements (TLE)

For monitoring by Norad *

```
ISS (ZARYA)
1 25544U 98067A   08264.51782528 -.00002182  00000-0 -11606-4 0  2927
2 25544  51.6416 247.4627 0006703 130.5360 325.0288 15.72125391563537
```

The meaning of this data is as follows:

LINE 1:

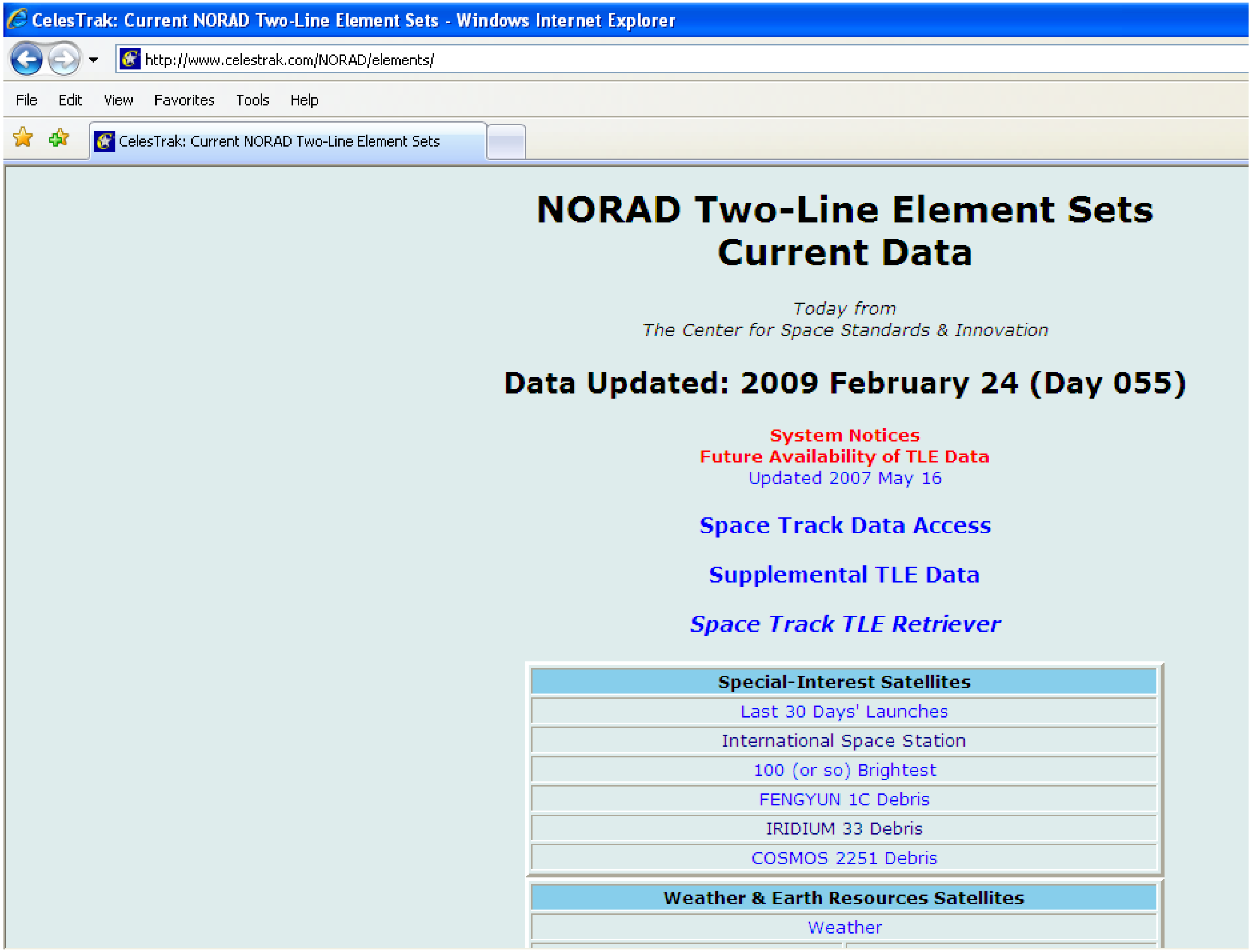
FIELD	COLS	CONTENT	EXAMPLE
1	01–01	Line number	1
2	03–07	Satellite number	25544
3	08–08	Classification (U=Unclassified)	U
4	10–11	International Designator (Last two digits of launch year)	98
5	12–14	International Designator (Launch number of the year)	067
6	15–17	International Designator (Piece of the launch)	A
7	19–20	Epoch Year (Last two digits of year)	08
8	21–32	Epoch (Day of the year and fractional portion of the day)	264.51782528
9	34–43	First Time Derivative of the Mean Motion	-.00002182
10	45–52	Second Time Derivative of Mean Motion (decimal point assumed)	00000-0
11	54–61	BSTAR drag term (decimal point assumed)	-11606-4
12	63–63	The number 0 (Originally this should have been "Ephemeris type")	0
13	65–68	Element number	292
14	69–69	Checksum (Modulo 10)	7

LINE 2:

FIELD	COLS	CONTENT	EXAMPLE
1	01–01	Line number	2
2	03–07	Satellite number	25544
3	09–16	Inclination [Degrees]	51.6416
4	18–25	Right Ascension of the Ascending Node [Degrees]	247.4627
5	27–33	Eccentricity (decimal point assumed)	0006703
6	35–42	Argument of Perigee [Degrees]	130.5360
7	44–51	Mean Anomaly [Degrees]	325.0288
8	53–63	Mean Motion [Revs per day]	15.72125391
9	64–68	Revolution number at epoch [Revs]	56353
10	69–69	Checksum (Modulo 10)	7

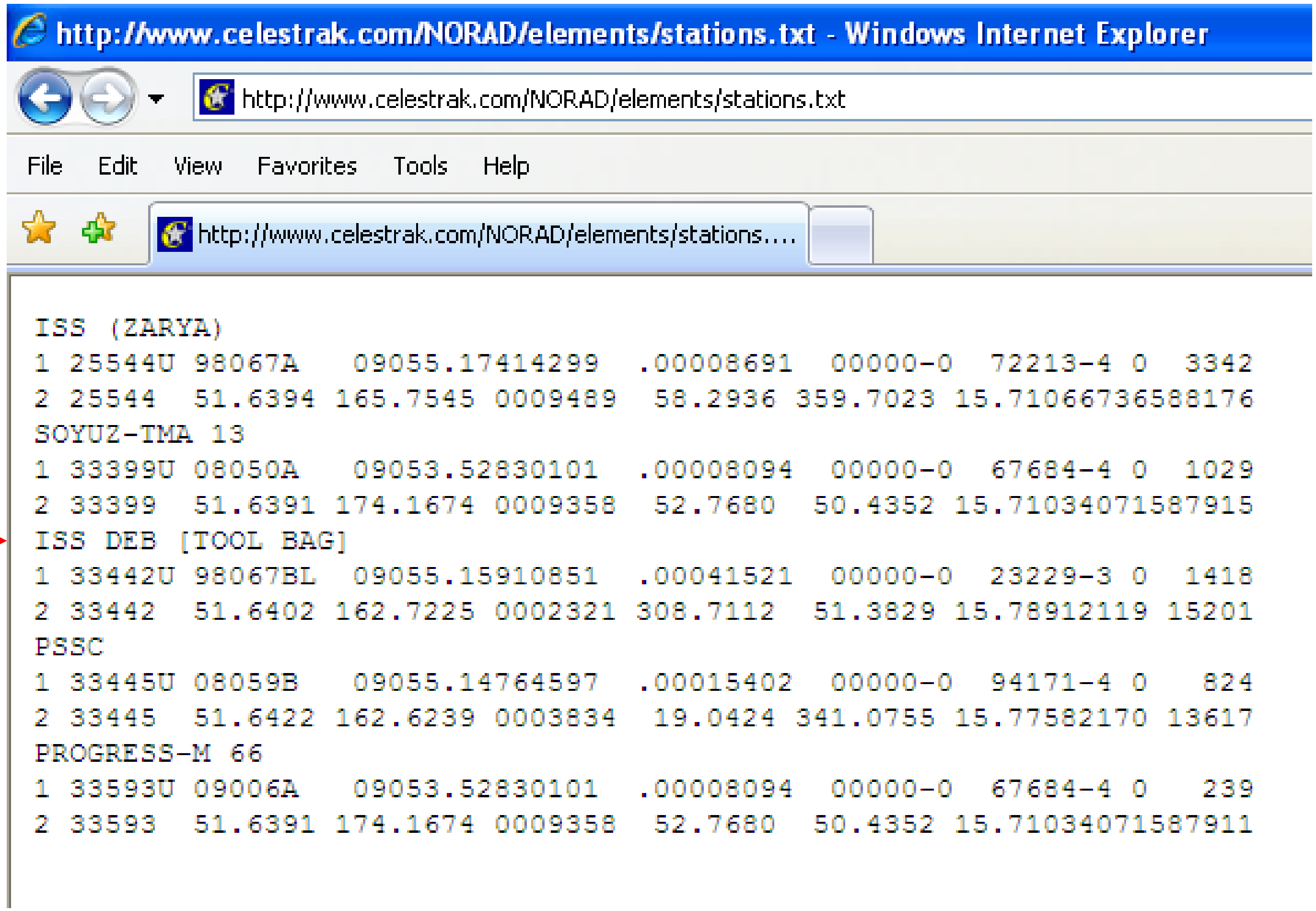
* North American Aerospace Defense Command

Celestrak: Update TLE



http://www.celestrak.com/NORAD/elements/

Celestrak: ISS, February 24, 2009

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2 25544  51.6394 165.7545 0009489  58.2936 359.7023 15.71066736588176
SOYUZ-TMA 13
1 33399U 08050A   09053.52830101  .00008094  00000-0  67684-4 0  1029
2 33399  51.6391 174.1674 0009358  52.7680  50.4352 15.71034071587915
ISS DEB [TOOL BAG]
1 33442U 98067BL  09055.15910851  .00041521  00000-0  23229-3 0  1418
2 33442  51.6402 162.7225 0002321 308.7112  51.3829 15.78912119 15201
PSSC
1 33445U 08059B   09055.14764597  .00015402  00000-0  94171-4 0   824
2 33445  51.6422 162.6239 0003834  19.0424 341.0755 15.77582170 13617
PROGRESS-M 66
1 33593U 09006A   09053.52830101  .00008094  00000-0  67684-4 0   239
2 33593  51.6391 174.1674 0009358  52.7680  50.4352 15.71034071587911
```

https://www.youtube.com/watch?v=1vXdRUIZ_EM

Lost ISS Toolbag

Heidemarie Stefanyshyn-Piper

From Wikipedia, the free encyclopedia

Heidemarie Martha Stefanyshyn-Piper (born on February 7, 1963
experienced salvage officer. Her major salvage projects include de-
Peruvian submarine Pacocha.

Stefanyshyn-Piper has received numerous honors and awards, such
115 and STS-126, during which she completed five spacewalks tota

Contents [hide]

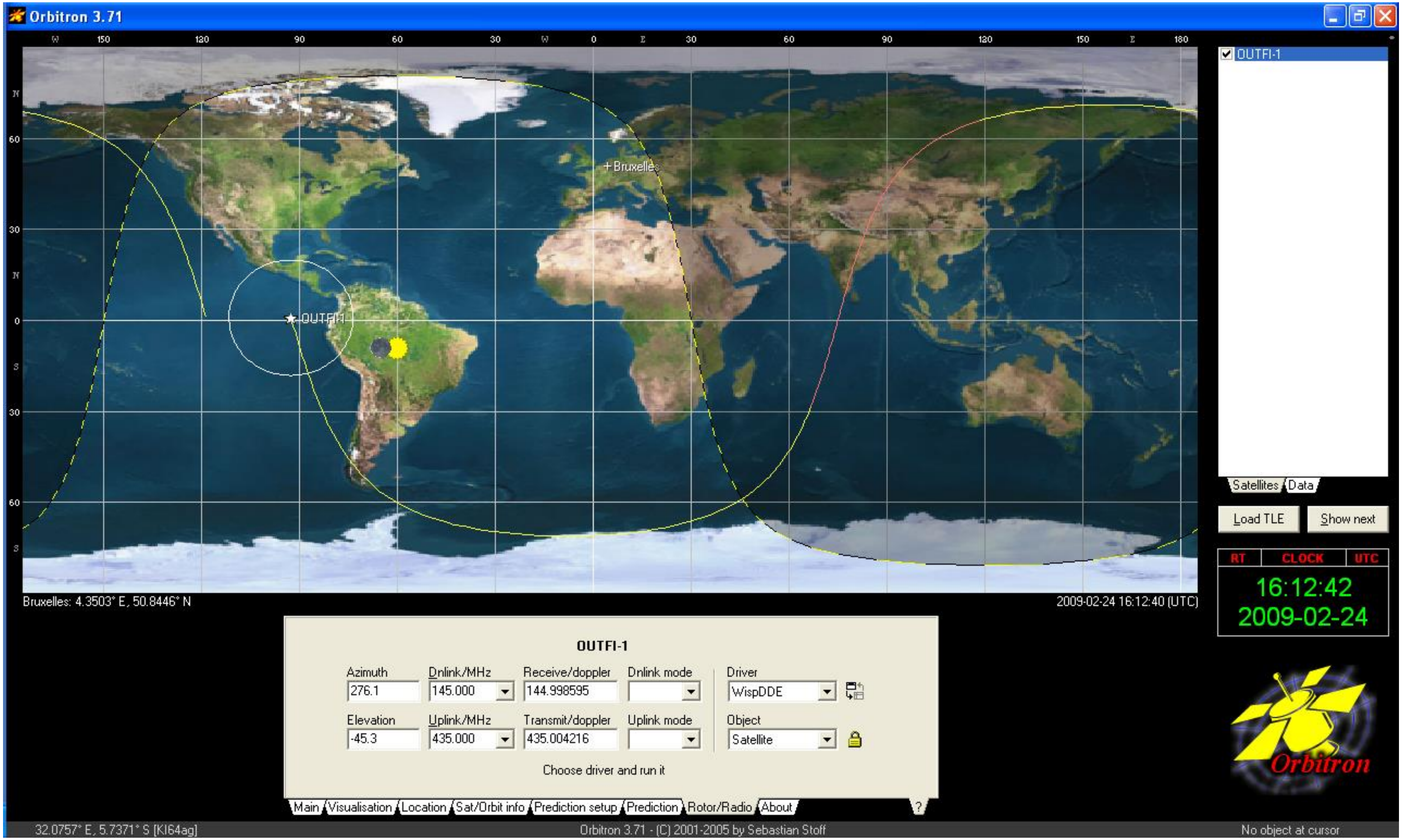
Cartesian ↔ Spherical

$$\mathbf{r} = X\hat{\mathbf{I}} + Y\hat{\mathbf{J}} + Z\hat{\mathbf{K}} = r\hat{\mathbf{u}}_r$$

$$\Updownarrow$$

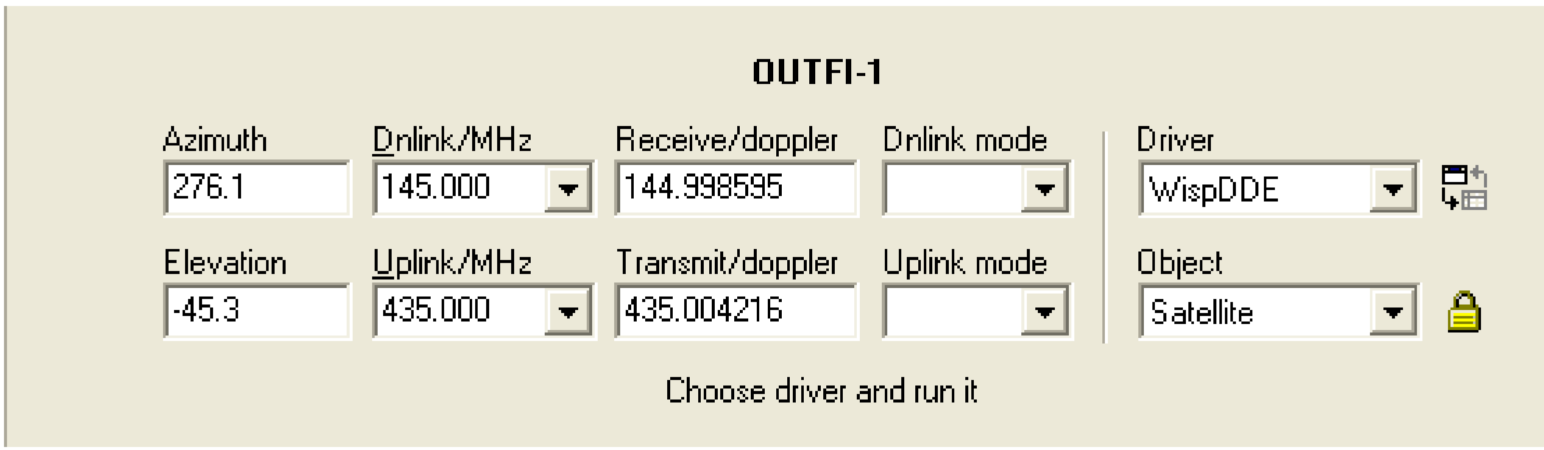
$$\hat{\mathbf{u}}_r = \cos\delta\cos\alpha\hat{\mathbf{I}} + \cos\delta\sin\alpha\hat{\mathbf{J}} + \sin\delta\hat{\mathbf{K}}$$

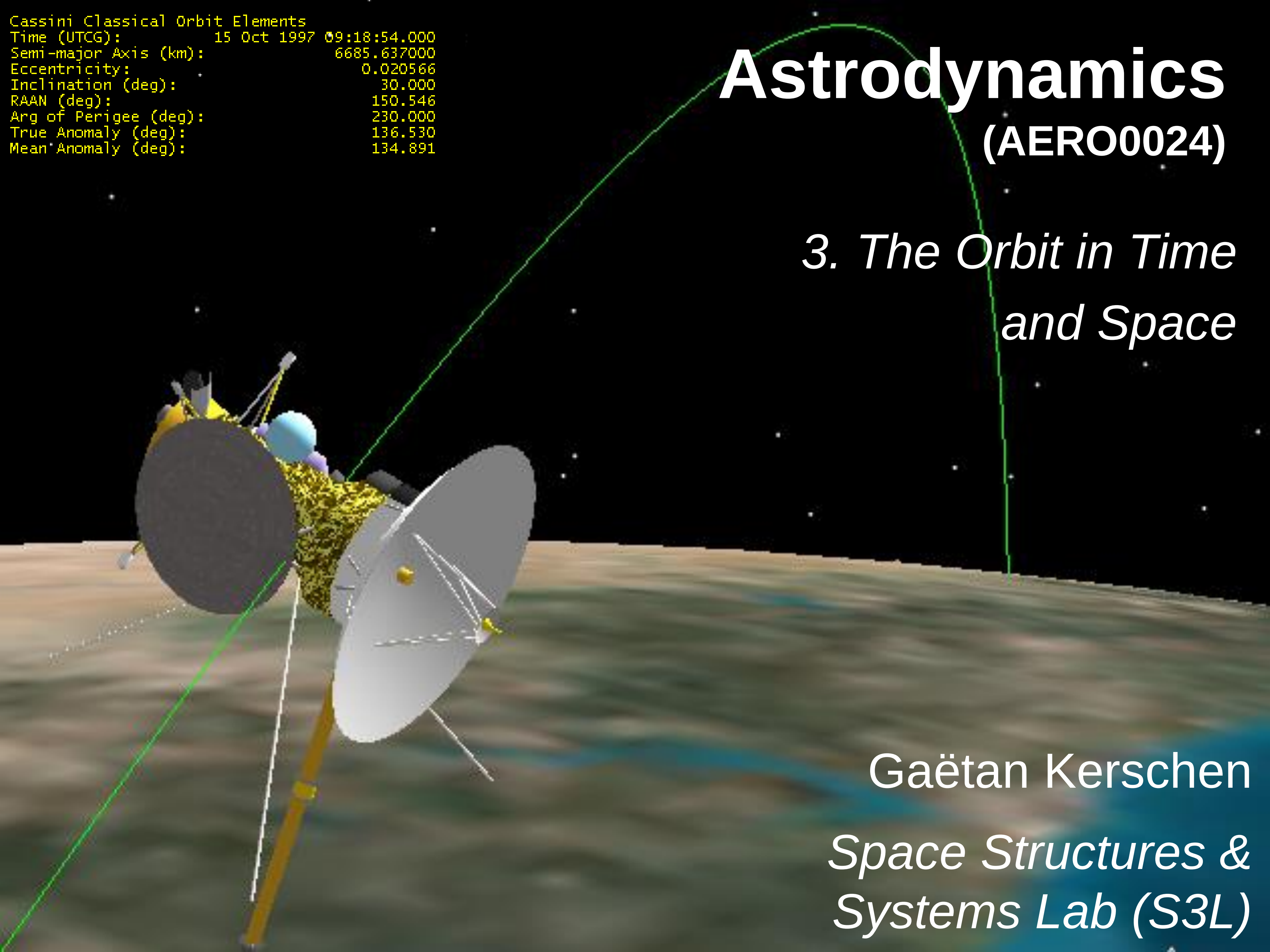
Orbitron



http://www.stoff.pl/

Orbitron: Close-Up





Astrodynamics

(AERO0024)

3. The Orbit in Time and Space

Gaëtan Kerschen

Space Structures & Systems Lab (S3L)